CRREL LEVEL

REPORT 79-4

Computer modeling of atmospheric
ice accretion

Cover: Accretion of rime ice, formed from sub freezing cloud droplets, on a suspended cable on the summit of Mt. Washington, New Hampshire, in April 1978. The cable was instrumented to measure the additional loading placed on it by wind and gravity forces acting on the attached ice. (Photograph by Jon Lingel, Mt. Washington Observatory.)

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S.F. Ackley and M.K. Templeton

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A computer model is described to compute the amount of ice accretion on an object under a variety of initial conditions. Numerical techniques are best applied to these problems because of time dependent effects governing the amount of ice collected and also the variety of initial conditions that can lead to ice accumulation. The helicopter rotor icing problem adds an additional complexity since the velocity along the rotor blade varies over a wide range strongly affecting the amounts of ice collected at different blade positions. The physics of ice accretion is reviewed and the accounting for time-dependence in the computer model is described. Some model results are presented and indicate the dependence of ice accretion on velocity, droplet sizes, cloud liquid water content, and temperature for a cylindrical object of constant size.

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PREFACE

This report was prepared by S.F. Ackley, Research Physicist, Snow and Ice Branch, Research Division and M.K. Templeton, Math Aide, Snow and Ice Branch (now at California Institute of Technology).

The study was conducted under DA Project 4A161102AT24, *Adhesion and Physics of Ice* and DA Project 4A7670AT42, *Prevention and Control of Ice Adhesion*.

Mr. Walter B. Tucker III and Mr. L. David Minsk reviewed the technical content of this report.

The authors wish to thank Dr. G. Ashton, Chief, Snow and Ice Branch and Program Manager for CRREL's Ice Adhesion Program for his continued support and encouragement of this work.

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INTRODUCTION

Work on ice accretion from the atmosphere has followed two distinct lines, hailstone growth and aircraft icing. Most recently, investigations of hailstone growth have provided the justification for a general understanding of ice accretion processes. The concept is that damaging hail grows in several regions within convective cumulus clouds. Through examination of the structure of hailstones, information on the ambient conditions within the clouds can be obtained, specifically liquid water content (lwc), droplet radius (R_{drop}) and air temperature (T_0). The hailstones can then be used to interpret and model cloud structures, and possibly to provide information on structure modification by cloud-seeding methods for preventing and slowing down hail growth. Macklin et al. (1976, 1977), List et al. (1971), and Knight and Knight (1968) provided the conceptual background for this application and described several recent efforts to obtain this information.

Early ice accretion work (e.g. Langmuir and Blodgett 1946) emphasized its application to airplane icing. Icing loads affected aircraft performance, making it necessary to provide ice detection and deicing systems (Tribus 1943, Messinger 1953). As aircraft evolved to types that flew higher and faster, however, research in this area declined, since major icing problems occur at altitudes below 20,000 ft (6100 m) and at air speeds below 300 mph (134 m/s), primarily in clouds ranging from about 0 to –20°C in temperature. The faster and higher flying commercial and military planes of today pass quickly through these conditions. Jet aircraft also have power available to overcome icing on engine intakes and other critical areas by heating, so recent efforts have been directed toward finding engineering solutions to typical or worst cases rather than toward continued research into the physics of the ice accretion process.

The fundamental physics indicated that particular cases could be formulated, but that the computations necessary to give a complete analysis were complex. For example, cumbersome analog devices requiring several persons working simultaneously were necessary

to obtain the initial water droplet trajectories of impaction. These initial trajectories were, of course, only one aspect of the total problem (Brun 1957, Langmuir and Blodgett 1946).

Research on helicopters and the possibility of making them capable of operation in any weather, including icing conditions, has proceeded sporadically but with continued interest since helicopter flight is limited mainly to the lower altitudes and air speeds where icing is more frequently encountered. Stallabrass (1957, 1958) conducted experiments on helicopters hovering under various icing conditions, using a water spray stand to simulate the icing cloud. Using the formulation developed from earlier aircraft work, he found that the icing accretion rate on the main rotor blades could be simulated by applying the fixed velocity calculation developed by Messinger (1953) for fixed-wing aircraft to each segment along the rotor blade characterized by its velocity:

$$v_i = r_i \omega \qquad (1)$$

where ω is the angular velocity of the rotor blade and r_i is the distance from the center of the hub (axis of rotation) to the segment of interest. Good agreement was found between the experimentally obtained ice shapes and those predicted using the calculations from the earlier work on aircraft at a single velocity (Dickey 1952).

The aim of this report is to demonstrate a method for calculating the quantity of ice that will accrete on fixed-wing aircraft, helicopters, ships, power lines, radomes, towers and other structures under a given set of conditions.

Although much of the previous work has been fundamentally correct in formulating the time-dependent process of ice accretion, one of the problems has been the exclusion of time dependence in the application of the formalism. In order to estimate ice accretion mass confidently over a wide range of conditions, the influence of this time dependence must be accounted for.

In this paper, we reexamine the fundamentals of the ice accretion process, and point out those features in

the governing relationships for ice accretion that require special treatment because they implicitly contain time dependence leading to feedback effects. We then show how the time-dependent formalism is solved using numerical methods to more correctly account for these feedback effects, and include them in calculations of ice accretion on objects, given the atmospheric conditions and object parameters. The computer model is useful as a simulation tool to provide input for design purposes, such as aircraft and helicopter deicing design, or ground-based structural applications, such as power line or tower icing prediction, or as a research tool to examine the sensitivity of the icing process to variations in the input variables, either singly or in concert. These simulations may assist in the understanding of limited laboratory and field testing, lending some generality to the ice accretion processes observed there, and extending this information to applications where data are limited, for example, power line icing in remote locations, and helicopter icing in the extreme cases that are currently untested.

ICE ACCRETION PARAMETERS

Atmospheric ice accretion is caused principally by the freezing of supercooled water droplets. The droplets may freeze after they are carried to a collision with an object by airflow (ground-based structure), or as an object, such as an airplane or helicopter, passes through them. Supercooled water is necessary because ice particles alone do not stick on impact and, therefore, do not cause an ice accretion problem. Recently there have been some indications that mixed clouds, containing both ice particles and supercooled water droplets, can cause a serious ice accretion problem, especially for helicopters (Stallabrass, pers. comm.). This problem ostensibly occurs because the ice particles, in the presence of supercooled water, adhere to the surface of the object, and, because they are already frozen, do not require the heat transfer that an unfrozen droplet of similar size requires to compensate for the large latent heat of freezing. However, icing tunnel tests have indicated that the amount of sticking by ice crystals is lower than expected, indicating that mixed conditions may not be as critical as was first assumed.

In the first case, we will consider only water droplets, since the effects of mixed conditions have not been studied sufficiently to allow a quantitative formulation. The primary sources of icing conditions are clouds and fogs since the smaller droplets present in them can sustain substantial supercooling over a wide temperature range. Raindrops are larger and therefore cannot undergo the same degree of supercooling (Mason 1971). However, the formulation can be applied to clouds, fog, rain, sea spray, and spray of man-made origin, e.g. power plant cooling tower fog, as long as the correct parameters can be specified. The six parameters necessary for quantifying the problem can be conveniently divided into three properties of the atmosphere—the ambient temperature T_0, the liquid water content lwc, and the droplet radius distribution R_{drop} —and three of the accreting object—its cross-sectional diameter ($2a$), velocity u and shape, e.g. cylinder, airfoil, etc. The interaction of these parameters will be identified in the next section.

PHYSICS OF ICE ACCRETION

If we move an object through a supercooled cloud, the rate of ice accretion depends on two factors. First, it depends on the kinematic interaction between the object and the droplets in the cloud. This relation determines whether particular droplet sizes are captured. Second, it depends on the thermodynamic processes at the surface of the object. Here, a balance is obtained between the rate of heat release necessary to freeze all or part of the impinging water and the rate at which heat can be carried away from the surface into the flow field by the processes of convection, evaporation and radiation. We first consider the kinematic interaction.

1. Interaction between water droplets and flow field

Normally, the speeds at which icing occurs are such that the air around the object can be treated as an ideal, incompressible fluid. The velocity field exhibits continuity, and mathematically (Sokolnikoff and Redheffer 1966) satisfies the condition for the application of potential theory (i.e. $\nabla \times u = 0, \nabla \cdot u = 0$). The velocity can therefore be solved for at any point around the object by solution of a complex potential of the form:

$$F = \phi + i\psi \qquad (2)$$

where ϕ is the velocity potential and ψ the stream function, and the curves of ϕ, ψ are orthogonal functions defining the complex flow field. The velocity of the fluid at any point is given by
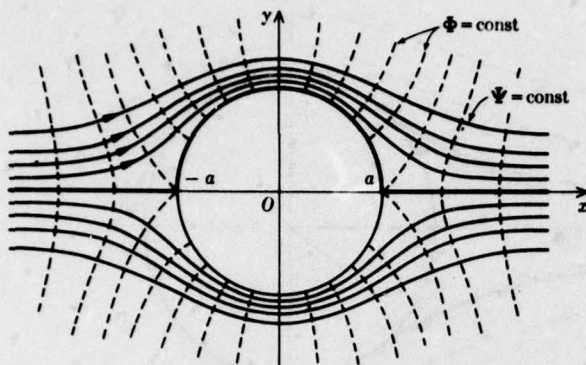
$$u_0 = \nabla\phi. \qquad (3)$$

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Figure 1. Velocity potential ϕ and stream function ψ around a circular cylinder of radius a embedded in a fluid with free stream velocity u.

Because of the orthogonality between ϕ and ψ, the velocity is tangent to the curves ψ = constant. These are the paths or streamlines of the fluid elements. We consider a circular cylinder as an example. Figure 1 shows the streamlines around such a circular cylinder, with flow perpendicular to the long axis. Two consequences of complex potential flow theory are that the boundary of the object coincides with one of the streamlines (ψ = const. at the boundary), and the flow satisfies Laplace's equation, $\nabla^2 F = 0$, in the entire plane. For objects of simple cross-sectional geometry, such as circles or ellipses, the dimensionless velocity at any point can be determined straightforwardly if we know the free stream velocity u (i.e. at great distances from the object) and the characteristic dimensions of the object—for the cylinder, its radius a (Milne-Thomson 1960).

To determine the interaction between the droplets and the flow field, we integrate the equation of motion for the droplet in the flow field. Following Brun (1957), the assumptions for this formulation are: a) the streamlines determined for clear air are valid, i.e. there are not enough droplets to perturb the flow, b) the gravity forces are much less than the inertial forces and may be neglected, c) the pressure forces on a droplet are equivalent to those on an equal volume of air at the same location, and consequently may also be neglected because water has a much greater density than air.

The motion of the droplet, therefore, results primarily from the inertial and viscous drag forces as it drifts with the fluid streamline. The drag force is proportional to the velocity and the drag coefficient c_d that the droplet sees at any given time. Newton's second law in dimensionless form for the droplet (after Langmuir and Blodgett 1946) therefore becomes, for the circular cylinder case:

$$k \frac{d\overline{u}_d}{dt} = \frac{c_d\,Re}{24}\,(\overline{u}_d - \overline{u}_0). \qquad (4)$$

Here $\overline{u}_d$ is the dimensionless velocity of the droplet at any point, $\overline{u}_0$ the dimensionless streamline velocity at the same point, Re the Reynolds number = $2R_{drop}$ $\rho_{air}\,[(\overline{u}_d - \overline{u}_0)u]\,/\mu$, c_d the drag coefficient, and k the inertial parameter (analogous to mass in dimensionless form) = $2\rho_w R_{drop}^2\,(u/9\mu a)$.

The drag coefficient c_d of a spherical drop is also a function of Reynolds number, and its dependence is given by Beard and Pruppacher (1969) as:

$$\left.\begin{array}{ll} c_d = 1 + 0.102\,Re^{.955} & 0.2 < Re < 2 \\[2mm] c_d = 1 + 0.115\,Re^{.802} & 2 < Re < 21 \\[2mm] c_d = 1 + 0.189\,Re^{.632} & 21 < Re < 200 \end{array}\right\} \quad (5)$$

The calculation of droplet trajectories involves considerable computation time, since the equation of motion has to be integrated numerically with respect to time to fully assess the degree to which the droplet deviates from the fluid streamlines. Langmuir and Blodgett (1946) compiled these computations into curves of the collection efficiency E as a function of two dimensionless parameters, each depending on the free stream velocity, droplet radius and cylinder radius. Collection efficiency is defined as the distance from the centerline of the cylinder axis divided by the cylinder radius that a droplet can be and still be captured by the cylinder, i.e. the far field position of the droplet trajectory that is tangent to the cylinder (Fig. 2). It also refers to that fraction of the total possible droplets in the path of the object that collected. For example, a collection efficiency of 0.5 indicates that half the droplets of a given size would be collected in the cross-sectional area swept out by the cylinder in the flow. Under natural conditions there is usually a variety of droplet sizes, so the computations become lengthy since an equation of motion has to be integrated separately for each size category. The total collection efficiency is then the sum of the collection efficiencies
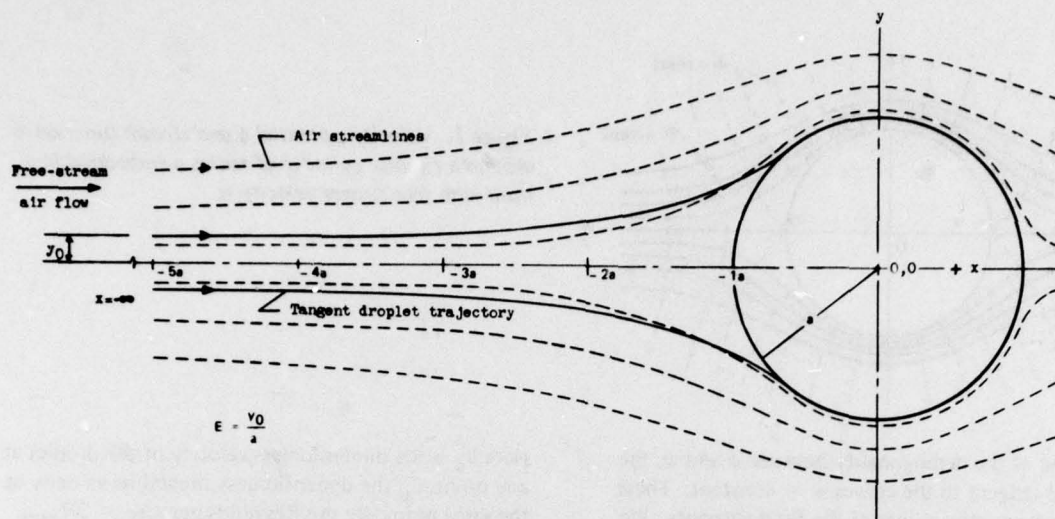
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Figure 2. Air streamlines and droplet trajectories with respect to a right circular cylinder. Collection efficiency E is the ratio of the initial far-field y location y_0 of the droplet trajectory that is just tangent to the radius of the cylinder a.

for each size times the fraction of the total *lwc* represented by that size.

2. Time dependence in droplet trajectories

The first of the time-dependent effects that have not been treated in detail in earlier work is that of droplet trajectory. As ice accretes on the front surface of the cylinder it is apparent that the cylinder cannot maintain its initial circular shape. Therefore, the streamline characterized by the boundary of the object is no longer the same as it was initially, and the velocity field in other regions is proportionally distorted to maintain continuity. From eq 5, the change in velocity of the droplet depends on the streamline velocity in that location. The trajectory and velocity of a droplet of a given size will therefore change with time from their values prior to ice accretion, thereby changing the collection efficiency E as ice accretes. This effect has not been included systematically before because of the length of the computations required to get the initial pre-icing values of the collection efficiency. With a digital computer of sufficient power, however, this effect can be taken into consideration as we show in a later section.

3. Thermodynamic processes at the surface of the object

In general, the mass rate of water arriving at the surface of the object (per unit length) can be written as:

$$\dot{m} = E \, lwc \, u \, (2a) \qquad kg/m\text{-}s \qquad (6)$$

where E is the collection efficiency, dependent on the velocity, the droplet size, and the object's cross-sectional diameter and characteristic dimensions; *lwc* is the volumetric liquid water content (kg/m^3); u is the free stream velocity (m/s); and 2a is the object's cross-sectional diameter normal to the flow (m). However, the mass of ice accreted will not in general be equal to this quantity if heat transfer processes cannot adequately freeze all of the accreted water.

In this formulation it is assumed that 1) radiation terms are excluded since they are small relative to convective and evaporative heat flux, 2) near the object, the droplets have the same velocity as air at that location ($\approx u_0$), 3) the droplets are at thermodynamic equilibrium at temperature T_0, the ambient air temperature, and 4) the boundary layer thickness is small so the droplet passes through the boundary layer and strikes the surface with the temperature T_0 and velocity u that it had in the flow. Recent work (List 1977, List et al. 1976, Joe et al. 1976) has indicated some problems with these assumptions, especially with the boundary layer interaction and the droplet's thermal state at high liquid water contents ($>0.002 \, kg/m^3$). But, as we indicate later, inclusion of different thermodynamics and assumptions can be accommodated in numerical solutions. We employ those just described because of their long usage and their verification, in general terms, by experimental observations.

The heat balance of the surface, controlled by the properties of the flow, is taken as the sum of the

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convective and evaporative fluxes, and the aerodynamic heating of the incoming air:

$$Q_{\text{surf}} = -H\left[(T - T_0) - \frac{Ru_0^2}{2c_{p\,\text{air}}} + \frac{0.6L_0}{pc_{p\,\text{air}}}(E' - E_0)\right]. \quad (7)$$

New terms are H, the convective heat transfer coefficient (defined below), R, the surface recovery factor, $c_{p\,\text{air}}$, the heat capacity of air, L_0, latent heat of water vaporization, E', vapor pressure of water at surface temperature T, E_0, vapor pressure of water at the ambient temperature T_0, and p, atmospheric pressure. The factor $0.6/p$ converts the mass flux from weights of water vapor to the more convenient vapor pressure form, and uses the relationship of latent and sensible heat transfer coefficients of $H_m/H \cong [F(\text{Le})/c_{p\,\text{air}}]$ where $F(\text{Le})$ [a function of the Lewis number Le (ratio of the diffusion coefficient to thermal diffusivity)] is approximately 1. Therefore, $H_m \cong (H/c_{p\,\text{air}})$, as indicated in eq 7.

The heat transfer coefficient per unit area, H_0, is taken from Bosch's formulation (Boelter et al. 1965):

$$H_0 = 31.01 \frac{u^{.56}}{(2a)^{.44}}\left(\frac{p \cdot 273.16}{1014F^*}\right)^{.56}. \quad (8)$$

Here F^* is the film temperature $= (T_0 + T)/2$. Initially the surface temperature T is taken as the recovery temperature $T = T_0 + (Ru_0^2/c_{p\,\text{air}})$ where $Ru_0^2/c_{p\,\text{air}}$ is the aerodynamic heating effect (in °C) from compression of the non-icing flow at the stagnation point. The heat transfer coefficient H is

$$H = H_0 A \quad (9)$$

where A is the surface area over which the identified processes are taking place, taken as the front half cylinder for our example.

The heat given to the blade by the incoming droplets (mass m per unit time) is dependent on whether none, some, or all of the water is accreted as ice, that is, the three cases of surface temperature: 1) $T>0°C$, 2) $T = 0°C$, 3) $T<0°C$.

Case 1. $T>0°C$

In this case, the heat flux is due to the temperature difference of the collected water plus the conversion of the droplet's kinetic energy into heat:

$$Q_1 = \dot{m}c_{p\,H_2O}\left(T - T_0 - \frac{u_0^2}{2c_{p\,H_2O}}\right). \quad (10)$$

Case 2. $T = 0°C$

In this case, a fraction F of the accreted water is converted to ice, adding the latent heat of fusion L to the heat flux:

$$Q_2 = \dot{m}c_{p\,H_2O}\left[(0 - T_0) - \frac{FL}{c_{p\,H_2O}} - \frac{u_0^2}{2c_{p\,H_2O}}\right]. \quad (11)$$

Case 3. $T<0°C$

Here all the accreted water is frozen as ice, i.e. $F = 1$ and the first term in the following equation accounts for the difference in specific heat between the water and ice at temperature T:

$$Q_3 = \dot{m}c_{p\,H_2O}\left(T\frac{c_{p\,\text{ice}}}{c_{p\,H_2O}} - T_0 - \frac{L}{c_{p\,H_2O}} - \frac{u_0^2}{2c_{p\,H_2O}}\right). \quad (12)$$

The thermodynamic steady-state balance requires that the heat taken by the flow, Q_{surf}, is balanced by the appropriate value Q_1, Q_2 or Q_3. In order to choose between these cases, Brun (1957) and Messinger (1953) have taken the heat balance for Case 2 and solved for the frozen fraction F. If F is between the limits 0 and 1 then Case 2 is the appropriate one. If $F>1$ then Case 3 is applicable, and if $F<0$ then Case 1 applies.

In a similar manner, the heat balance as shown schematically in Figure 3 (after Messinger 1953) can be taken for any case, and the residual heat in a short time interval Δt

$$Q = (Q_{\text{surf}} - Q_i)\,\Delta t \qquad i = 1, 2, 3 \quad (13)$$

can then be used to modify the surface temperature by

$$T_{\text{new}} = \frac{Tc_{p\,m\cdot\text{blade}} + Q}{c_{p\,m\cdot\text{blade}}}. \quad (14)$$

Here

$$c_{p\,m\cdot\text{blade}} = c_{p\,m\cdot\text{blade}} + m\cdot F\cdot c_{p\,\text{ice}} \quad (15)$$

i.e. the total heat capacity of the object (blade) plus the heat capacity of any accreted ice. The new temperature T_{new} can then be checked to see if it is above or below 0°C and the appropriate thermodynamic branch (Q_1, Q_2, Q_3) selected. In going from above or below 0°C, the option $T = 0°C$ is selected and the appropriate balance chosen based on the value of the accreted fraction F. F must lie within the range $0<F<1$ in order for the $T = 0$ thermodynamics to be chosen.
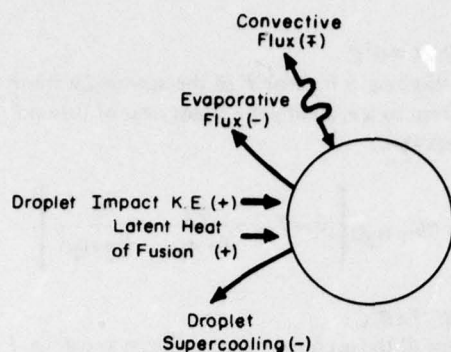
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Figure 3. *Schematic indicating the five terms used in the heat balance and whether they are carrying heat toward (+) or away (–) from the freezing surface. Convective flux usually takes heat away from the surface, but at high velocities adiabatic compression of the flow dominates and the convective heat flux is then positive toward the surface.*

4. Time dependence in the thermodynamics

As with the droplet trajectories, the time dependency of the thermodynamics associated with ice accretion basically results from the distortion of the object's size and shape as the ice accretes.

Equation 6 for the mass rate of water collected contains this dependence in the collection efficiency term, which changes as the flow field responds to the increasing accretion. The heat flux from the surface into the flow (eq 7) varies with the change in heat transfer coefficient (eq 9), which in turn varies through the changes in surface area and shape with time. Since the heat fluxes Q_i additionally depend on the mass rate of water accreted (eq 6), they are also time-dependent in the same manner as the mass rate (i.e. in the collection efficiency E). In general, then, the correct form of the equations exhibits significant time dependence. A scheme to update the time-dependent parameters as ice accretion proceeds is described later.

In summary, the major changes in the characteristic manner in which ice accretes on a given object with time are: the variation in the flow field (or collection efficiency) that affects the subsequent mass rate of collection, an increase in surface area that affects the total heat transfer coefficient, and, to a lesser degree, changes in the heat capacity of the object through the addition of ice.

5. Time dependencies of lesser order

Ice density

The effects described here are less significant but can be quantified to some degree and may be worth-while including in a detailed description of the icing process. The first of these is the ice density. Macklin (1962) found that in relatively low velocity flows and for surface temperature below 0°C (Case 3 above), the ice density could be obtained as follows:

$$\rho_{ice} = 0.110 \left(\frac{10^4 R_{drop} u_0}{ABS\ T} \right)^{.76} \quad (Mg/m^3). \quad (16)$$

For ice formed either near the melting point or at higher velocities, or for larger droplets, the density will approach the theoretical maximum density of 0.917 Mg/m³ and eq 16 will no longer apply. This variation may induce a time dependence since the volume of the ice accretion, as well as its mass, will change the subsequent flow field. The volume can be determined by the mass deposited (eq 6) and the density as given in eq 16.

Run-back icing

For Case 2 ($T = 0$), only a fraction of the accreted water is initially frozen. In our first formulation we assumed that the unfrozen water is carried away by the flow after imparting its supercooling to the surface and affects neither the thermodynamic processes nor the flow field. Observations during flight tests and on experiments have indicated, however, that icing has occurred on the rear portions of objects as this water flowed back over the surface and refroze. This formation is known as "run-back icing." It is a difficult parameter to include because of the formation conditions—how the water is maintained on the surface and whether heat transfer is rapid enough to freeze it before it is partially or totally removed as liquid at the trailing edge. Cansdale and McNaughton (1977) and Stallabrass and Lozowski (pers. comm.) are currently including a run-back formulation to determine angular dependence of initial icing rate without time-dependence. It appears an extension of the present work with this angular dependent thermodynamics could be a significant step forward in our understanding of the accretion process.

Spongy ice growth

Spongy ice is formed by a mesh of interlocking dendrites that trap a fraction of the unfrozen water and cause it to freeze by subsequent heat removal, possibly through conduction processes to the side and rear surfaces of the object on which ice is accreting. Spongy ice is not expected to occur frequently under flight conditions because the flow would mechanically disrupt the fragile structure that this ice has when it first begins to form. However, at low velocity (<10 m/s, List 1977) or low Reynolds number, there are indications that spongy ice would exist.
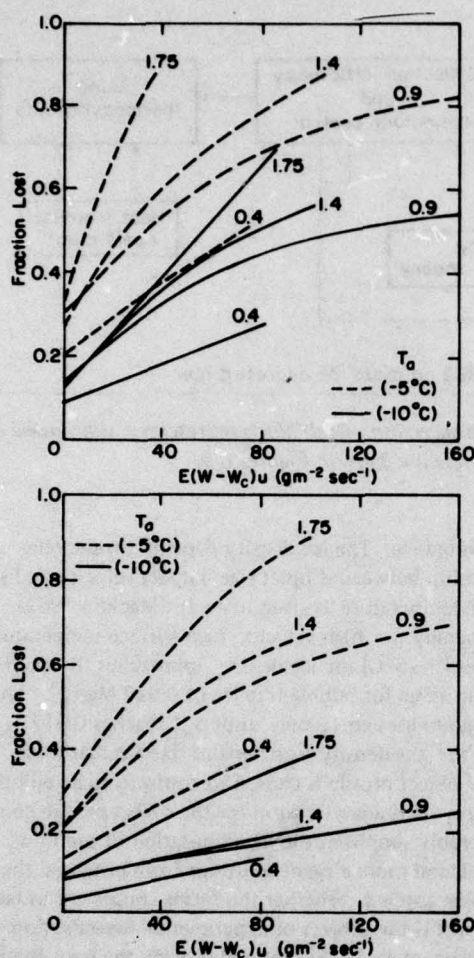
Figure 4. The fraction of accreted water lost plotted against $E(W - W_c)u$ for various temperatures and average radii (cm) of accreting objects (after Carras and Macklin 1973).

In a previous paper (Ackley 1977) we described how the spongy growth can be incorporated into a full time-dependent solution for icing prediction. Here we briefly describe the variations from the thermodynamics detailed in this report.

The standard thermodynamics are first evaluated to determine that the $T = 0°C$ case exists. Instead of a frozen fraction F depending only on the available heat transfer, empirical relations are used to determine the fraction of the accreted water that is lost to the flow as shown in Figure 4. The quantity $(W-W_c)u$ is plotted against the fraction lost, W and W_c representing the total /wc and frozen /wc respectively (i.e., W_c corresponds to that amount of /wc that would raise the surface temperature to 0°C but still freeze all the

accreted water). $E(W - W_c)u$ represents the initially unfrozen fraction of the total accreted water, and the curves shown give the fraction lost for various temperatures and radii of the objects. The quantity (1-fraction lost) of initially unfrozen water is that which contributes to the total ice accretion through the spongy ice phenomenon.

In the numerical model, the spongy ice growth could be incorporated by defining a Reynolds number function that determines the changeover from the case where the unfrozen fraction is shed to the case where part of it is incorporated according to the relations shown in Figure 4. Experiments are necessary to verify such a condition since the thermodynamics are dependent upon geometry, the heat capacity of the material, and the characteristics of the flow field in order to freeze additional water over and above that determined for Case 2 ($T = 0$) and may be difficult to quantify.

The shape of the accretion is especially critical since the cross-sectional diameter ($2a$) can be modified by the spongy growth regime, as in the lobe structures seen on hailstones. These changes in shape lead to substantial changes in the subsequent mass rate of accreted water through the collection efficiency effect, the total surface area available for heat transfer, and other first-order feedback effects that are controlled by the cross section.

NUMERICAL ICE ACCRETION MODEL

1. Major subroutines

In a first attempt to include time dependence and its subsequent effects on later ice accretion, a numerical model of the ice accretion processes described has been programmed on a digital computer. The model is programmed in the language Basic on the Dartmouth Time Sharing System (DTSS), which uses a Honeywell 66/40 computer. Hook-up is through an acoustic coupler connected by telephone line to the computer. The model includes software using DTSS subroutines for graphic display of the results using a Tektronix Model 4013 CRT display. A listing of the program, called Ice9, is given in Appendix A. In Figure 5, the feedback effects of time dependence are indicated by the various arrows. The major one is the recomputation of the object's profile after ice accretes, based on the mass of accreted ice and the density variation. The new profile is then used to update the flow field and determines changes, for example in the collection efficiency E and heat transfer coefficient H, for the next time step.

Each block of Figure 5 corresponds to a subroutine of the program, and we will describe each of these, keeping in mind their overall interconnection as shown in Figure 5. The initial variables chosen are indicated in Figure 6. These have been described in earlier

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Initial variables → **Object profile calculation** → **Droplet trajectory calculation** → **Collection efficiency and trajectory control** → **Icing thermodynamics**

Ice density calculation

Free stream velocity via complex potential flow theory

Heat transfer coefficient

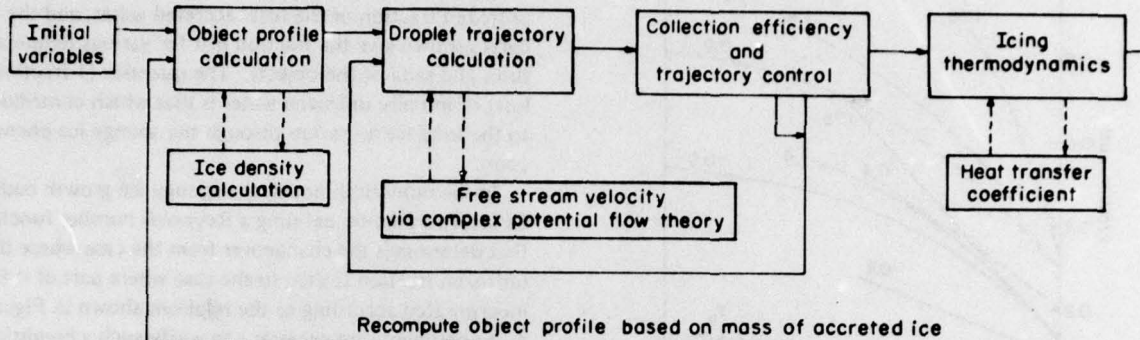Recompute object profile based on mass of accreted ice

Figure 5. Block diagram of the numerical model for calculating ice accretion. Each block represents a subroutine of the program. Flow diagrams and pertinent equations for each of these are given in Figures 6-9.

Initialize Variables

Air temperature = T_0

Liquid water content = lwc

Droplet radius = R_D
[or droplet radius distribution = $P(R_D)$]

Object airspeed = u
[or free stream air velocity]

Figure 6. Initial variables input in the run control sequence of the numerical model. Other parameters such as the object size and total time of the simulation can also be varied but these require program statement modifications (see Appendix A).

sections, and we need only mention here that the program is initially set for a circular cylinder of fixed radius and heat capacity, so these parameters are built in as program statements rather than initial variables. They can be varied, however, and comments within the program statements (see Appendix A) indicate the appropriate variables that refer to them. The choice of droplet sizes is controlled by a separate subroutine, Dropdst, which will be described later. The program is initially set to run by asking a series of questions concerning the selection of initial variables, so the choice of either a single droplet size or a number of sizes (e.g. mean and standard deviation for a Gaussian distribution) is automatically routed in the program.

In Figure 7a, the object's profile is recalculated at a time increment set for the given icing conditions. The first step is to take the mass of the icing accretion output by the thermodynamics and the subroutine for ice density (Fig. 7b) and compute the volume of the

accreted ice. The ice density depends on the relationship between droplet size, impact velocity and surface temperature as given in eq 16 (Macklin 1962). Generally, for high velocity, high surface temperature cases ($> -5°C$) the ice density approaches the maximum value for bubble-free ice of 0.917 Mg/m^3. The program uses eq 16 only until ρ_{ice} reaches 0.917 Mg/m^3; the density then remains fixed at that value. The new object profile is chosen to conform to an elliptical shape. This analytic form for the object profile considerably simplifies the recomputation of the flow field, and more elegant solutions from potential theory can be applied. Whether the forms chosen are in fact correct is the subject of experimental investigation (Ackley et al. 1978) that will provide the basis for a better estimate of the object profile. At present, the minor axis of the ellipse perpendicular to the flow direction remains constant at the initial value while the major axis parallel to the flow direction is increased in the direction of the accreting ice in relation to the mass and density of the accreted ice (e.g. see Fig. 11, 13, 14). Preliminary experiments indicate that these assumptions are not seriously in error, although observed small changes in cross section can markedly affect the results, since the cross section appears directly in both the mass rate and the thermodynamic equations.

Figure 8 shows the subroutines controlling the computation of the droplet trajectory, the collection efficiency and trajectory control, and the free stream velocity. The subroutine Traject (Fig. 8a) integrates the equation of motion for the droplet, calling upon the subroutine Velocity (Fig. 8c) as needed to determine the velocity of the flow field at each point on the droplet's path, and updates the droplet's position until one of the three cases of interaction with the object is determined. In order to compute the collection

8

Object profile calculation
[PROFILE]

1 Call: subprogram ICE DEN to find density of accreted ice

2 The total volume of ice accreted in $cm^3/(meter\ of\ blade/2)$

$$V = \frac{m\,F\,1000\ g/kg}{\rho_{ice}}$$

3 This changes the cross sectional area by

$$V' = \frac{V}{100\ cm}$$

4 New major axis length:

$$a = \frac{4V'}{b^2\pi} + a$$

a.

Ice density calculation
[ICE DEN]

1 $\rho_{ice}\ (Mg/m^3) = .110[10^4\,R_{drop}\,u_0/ABS(T)]^{0.76}$

(Macklin, 1962)

b.

Figure 7. Subroutines Profile and Ice Den used to calculate the changes in object profile and ice density from the output of the icing thermodynamics (Thermo, Fig. 9a).

efficiency for a particular droplet size category, the trajectory that is just tangent to the object must be determined. Therefore, the three cases of the droplet's path are a) where the droplet impacts the object but not at a tangent, b) where the droplet misses the object, and c) where the trajectory is tangent. The interaction status, along with other parameters of this trajectory, is then fed to the Coll Eff subroutine. If the trajectory is not tangent to within an arbitrary amount, this subroutine provides Traject with a new "intelligent" guess for the off-axis starting position of a droplet that will produce a tangent trajectory. Thus Coll Eff serves as the control for Traject, and the program switches between the two subroutines until a tangent interaction is found. Coll Eff then assigns the collection efficiency for the droplet size defined as the starting off-axis (y-value) for the path that ends up tangent to the object, as a fraction of the object's cross section. When

all droplet sizes have been accounted for, basically by the droplet distribution subroutine (Dropdst), the program takes the calculated collection efficiencies and advances into the icing thermodynamics.

Figure 9a shows the subroutine Thermo. This routine takes the output of the collection efficiency routine and the initial variables and computes the mass of ice accreted, based on the thermodynamic balance. The temperature of the object's surface is initialized in the control program at the recovery temperature of a non-icing flow at the same velocity.

The model proceeds through the icing thermodynamics based on the pseudo steady-state temperature hypothesis, i.e. over a small enough increment of time the temperature of the object does not change. So, before the thermodynamic balance is initiated, the heat transfer coefficient is calculated in the subroutine Httrans1 (Fig. 9b), using the current value of the surface

Droplet trajectory calculation
[TRAJECT]

1. Calculate k, (inertial parameter) $k = \lambda_s/a = 2\rho_w R_{drop}^2 u/9\eta a$
   (Langmuir and Blodgett, 1946)

2. Calculate Reynolds number of the droplet moving with respect to the air ◄———

3. Calculate the drag coefficient c_D ($c_D Re/24 = f/6\pi\eta u_d R_{drop}$):
   Beard and Pruppacher (1969) give

   $$c_D = 1 + 0.102\ Re^{.955} \qquad 0.2 < Re < 2$$

   $$c_D = 1 + 0.115\ Re^{.802} \qquad 2 < Re < 21$$

   $$c_D = 1 + 0.189\ Re^{.632} \qquad 21 < Re < 200$$

4. Call: subprogram VELOCITY for calculating free stream velocity of air
   at droplet's current location

5. a) $du_d = c_D\ Re/24\ (u_d - u_0)\ (dt/k)$ (change in velocity of droplet)

   b) $u_d = u_d + du_d$ (new velocity of droplet)

   c) $x_d = x_d + u_d\ dt$ (new position of droplet)

6. Is the droplet downstream far enough for it to be capable of hitting the
   profile, i.e. is

   $$x_d \leq a$$ No   Reiterate ———►

7. Compare the tangent profile and the droplet trajectory tangent (v_0/u_0)
   if $|\tan_{profile}| < |(v_0/u_0)|$ No
   yes, then particle (could have) missed profile

8. Check to see by how much droplet missed profile

   $$\text{if}\ \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) - 1 < .01$$

   then droplet missed profile, but the trajectory was $\approx$ tangent ———

9. ** missed profile ** let $H_0 = 1$
   end trajectory

10. Did we hit target yet? $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 - 1 < 0$ No   Reiterate ———

11. Was the trajectory within .1 arctan units of being tangent to the profile
                                No ———

12. ** tangency found **   Let HO = 2 ◄
    end trajectory

13. Let HO = 3
    ** incorrect hit **

*a. Subroutine Traject uses the object profile output (Fig. 7a) and calls the subroutine
Velocity to calculate the free stream velocity (Fig. 8c). It then integrates the equation
of motion to determine whether and how a droplet of particular size impacts the ob-
ject. The HO values are used to indicate the droplet has missed the object (HO = 1),
hit the object at a tangent (HO = 2), or hit the object but not at a tangent (HO = 3).*

*Figure 8. Subroutines controlling computation of the droplet trajectory, collection
efficiency and trajectory control, and free stream velocity.*

Collection efficiency calculation and trajectory control
[COLL EFF]

HO = 1
HO = 2
HO = 3

1 (HO = 3) Incorrect hit

$$\text{new } y_0 = y_0 \left(\frac{y_T}{y_i}\right)$$

$$y_T = b[1 - (\tau/a)^2]^{1/2}$$

$$\tau = ABS\left[\frac{a^2 v_0/u_0}{\left[b^2 + \left(a\frac{v_0}{u_0}\right)^2\right]^{1/2}}\right]$$

$$y_i = (v_0/u_0)\,\tau + y - (v_0/u_0)\,x$$

2 if the correction < .001 then ≈ tangent   yes

No

3 tangent to profile at point of impact

*** assign collection efficiency, E ***

Let $E = y_0$

(normalized to object axis b)

TRAJECT

4 (HO = 1) Target miss

$$\text{new } y_0 = y_0\,(y_T/y_p)$$

5 if the correction < .001 then ≈ tangent

No

Extrapolated trajectory (st. line)
Actual trajectory
(x,y)
y_0
y_T
y_i

y_p
y_0
y_T
4

b. Subroutine Coll Eff takes the droplet trajectory information to compute the collection efficiency E for the given droplet size and object profile. If the correct tangency value of droplet impact is not found, the subroutine readjusts the droplet's initial off-axis position (y_0) and calls Traject to compute the correct tangent trajectory. Once the collection efficiency is assigned, the program proceeds to Thermo (Fig. 9a).

Figure 8 (cont'd).

temperature. Then, based on the current surface temperature, the appropriate thermodynamic expression is chosen (Fig. 9a). The complete thermodynamic balance yields the new surface temperature, or new ice fraction accreted if in the $T = 0°C$ temperature regime. Of course, care must be taken to maintain physical as well as thermodynamic continuity as the temperature proceeds through the limits of each thermodynamic argument. The program checks for errors in continuity and redirects execution at the appropriate level.

At the end of the icing thermodynamics, the output

is given as new ice mass, total heat capacity, and surface temperature. After several iterations of the thermodynamics, the new total ice mass is fed back to the object profile calculation (Fig. 5, 7) to recompute the object's size, flow field characteristics, and surface area for the next time period of the calculation.

2. Options for droplet size variations

In addition to the basic computational portions just discussed, the Dropdst subroutine allows the specification of more than one droplet size for any given simulation. Dropdst has three options available on

Stream velocity
[VELOCITY]

From complex potential flow theory the velocity at any point x, y in the flow around an elliptical cylinder is given by:

$$u_0 = \frac{u(a+b)}{a^2 - b^2}\left[\frac{bR}{\alpha}\cos(\theta - \beta) - a\right]$$

$$v_0 = -\frac{u(a+b)}{a^2 - b^2}\left[\frac{bR}{\alpha}\sin(\theta - \beta)\right]$$

where:

$$\alpha = [(x^2 - y^2 - c^2)^2 + 4x^2 y^2]^{1/4}$$

$$\beta = \frac{1}{2}\tan^{-1}\left[\frac{2xy}{x^2 - y^2 - c^2}\right]$$

$$R = (x^2 + y^2)^{1/2} = x_d$$

$$\theta = \tan^{-1}(y/x)$$

a = semimajor axis length ($\parallel u$)

b = semiminor axis length ($\perp u$)

see *Milne-Thomson*, 1960

c. Subroutine Velocity computes the velocity in the flow at a given x,y position based on potential flow theory. Traject (Fig. 8a) uses this information in integrating the equation of motion and computing the droplet's trajectory.

Figure 8 (cont'd). Subroutines controlling computation of the droplet trajectory, collection efficiency and trajectory control, and free stream velocity.

droplet distributions in addition to the single droplet size, which does not require the calling of the Dropdst subroutine. Currently the droplet distributions available are a Gaussian, an Erlang of order 1, 2 or 3, or a histogram of sizes that could be, for example, from experimental data. In running the program, the operator is asked to specify whether a distribution is desired and, if so, this subroutine is called. It then asks additional questions concerning the type of distribution desired and the necessary parameters for specification of this distribution.

Gaussian distribution
The program asks for the mean, radius variance and total number of droplet sizes desired up to a maximum

of nine sizes. The subroutine then apportions the available liquid water content (specified initially in the control sequence) into these categories in such a way that the total number of droplets conforms to the distribution as specified. The Gaussian is sampled between ± 2 standard deviations of the mean so that it includes 95% of the distribution.

Erlang distribution
The Erlang distribution is of the form

$$F_r(r - r_0) = \frac{\lambda^k (r - r_0)^{k-1} e^{-\lambda(r - r_0)}}{(k-1)!} \tag{17}$$

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Icing thermodynamics
[THERMO]

1 Calculate the ambient water vapor pressure (E_0)
    Smithsonian Meteorological Tables, 1951

2 Call: subprogram HTTRANS1 for calculating heat transfer coefficient
    [watts/°C (meter of blade/2) sec]

3 Calculate mass of water collected/meter of blade · sec

$$m = E \cdot lwc \cdot u \cdot b$$

4 $\left.\begin{array}{l} T < 0 \\ T = 0 \\ T > 0 \end{array}\right\}$ branch to appropriate thermodynamics
    (see E.A. Brun, 1957)

$\left[\begin{array}{l}\text{initially,} \\ T = T_0 + \dfrac{u_0^2}{2000} \\ \text{i.e. recovery temp}\end{array}\right]$

a. Subroutine Thermo calculates mass of water arriving and converted into ice or shed as unfrozen water if $T > 0°C$.

5 $T > 0$ what is Q?

Figure 9. Based on the ambient conditions, the calculation of mass accreted is completed by incorporating the thermodynamics at the surface.

a) $Q = -H\left[T - T_0 - \dfrac{Ru_0^2}{2c_{p\,air}} + .6\,L_0(E' - E_0)/p\,c_{p\,air}\right] -$

$$- m\,c_{p\,H_2O}\left(T - T_0 - \dfrac{u_0^2}{2c_{p\,H_2O}}\right)$$

R = surface recovery factor
L_0 = latent heat of vaporization of water @ 0°C
E' = vapor pressure of water at surface temperature

b) Calculate new T

$$T = (Tc_{p\,m\,blade} + Q)/c_{p\,m\,blade}$$

c) Is T still above 0°C?   No
   ** end THERMO **

6 $T = 0$

a) $Q = -H\left[-T_0 - \dfrac{Ru_0^2}{2c_{p\,air}} + .6\,L_0(E' - E_0)/p\,c_{p\,air}\right] -$

$$- m\,c_{p\,H_2O}\left(-T_0 - \dfrac{FL}{c_{p\,H_2O}} - \dfrac{u_0^2}{2c_{p\,H_2O}}\right)$$

F = fraction of collected water accreted as ice
L = latent heat of freezing

b) $F = F - Q/Lm$

c) is $F > 0$ Temperature above 0°C   No

d) is $F < 1$ Temperature goes below 0°C   No

e) $c_{p\,m\,blade} = c_{p\,m\,blade} + mF\,c_{p\,ice}$
   ** end THERMO **

7 $T < 0$

a) $Q = -H\left[T - T_0 - \dfrac{Ru_0^2}{2c_{p\,air}} + .6L_0(E' - E_0)/p\,c_{p\,air}\right] -$

$$- m\,c_{p\,H_2O}\left(T\dfrac{c_{p\,ice}}{c_{p\,H_2O}} - T_0 - \dfrac{L}{c_{p\,H_2O}} - \dfrac{u_0^2}{2c_{p\,H_2O}}\right)$$

b) $T = (Tc_{p\,m\,blade} + Q)/c_{p\,m\,blade}$

c) $c_{p\,m\,blade} = c_{p\,m\,blade} + m\,c_{p\,ice}$
   SUBEND

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Heat transfer coefficient
[HTTRANS1]

$$1 \quad H_0 = 31.01 \, u_0^{0.56}/(2a)^{.44} \left(\frac{p * 273.16}{1014 \, F*} \right)^{.56} \quad \frac{\text{watts}}{\text{m}^2 \, ^\circ\text{C sec}}$$

$$F* = \text{film temperature} = \frac{T_0 + T}{2}$$

$$2 \quad \text{the total heat transfer coefficient} \Big/ \frac{\text{meter of blade}}{2}$$

$$H = H_0 \, \pi \, \frac{a + b}{4} \left(\frac{1}{100} \right)$$

b. Subroutine Httrans1 computes the total heat transfer coefficient based on the current surface temperature and object dimensions. It is called by Thermo (Fig. 9a) as needed.

Figure 9 (cont'd). Based on the ambient conditions, the calculation of mass accreted is completed by incorporating the thermodynamics at the surface.

$$F_r(r - r_0) = \frac{\lambda^k (r - r_0)^{k-1} e^{-\lambda(r-r_0)}}{(k-1)!}$$

$$\bar{r} = \frac{k}{\lambda} + \bar{r}_0$$
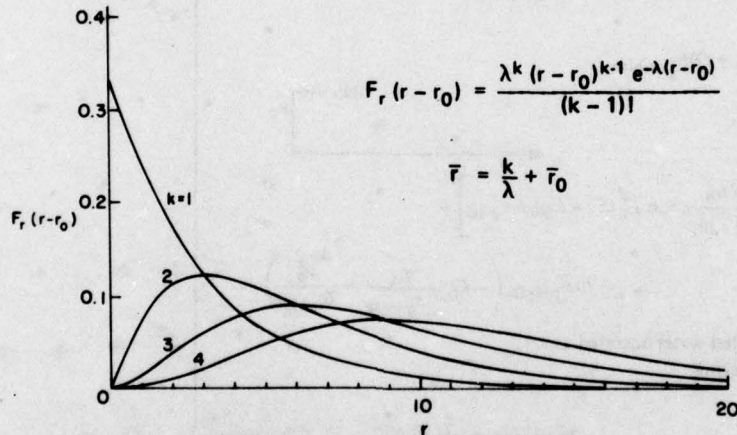
Figure 10. Forms of the Erlang distribution for various values of the parameter k as shown. The "start radius" r_0 is zero for these cases.

and has the forms shown in Figure 10 for various values of the "Erlang order" k and for "start" values $r_0 = 0$. The mean or expected value of the distribution is

$$\bar{r} = \frac{k}{\lambda} + r_0. \tag{18}$$

As Figure 10 shows, a variety of distributions can be obtained by specification of a few parameters

(k, λ, r_0), making it easy to change the characteristics of the distribution for simulation comparisons. The distributions are also more reminiscent of the variety one sees in real cloud droplet distributions than the Gaussian distribution (Mason 1971). The subroutine asks the specification of the values of $k, \bar{r}, r_0$ and the number of droplet classes, up to a maximum of nine, that are desired. The subroutine then calculates the value of λ from eq 18 and apportions the available *lwc* into the specified classes according to eq 17. The

14
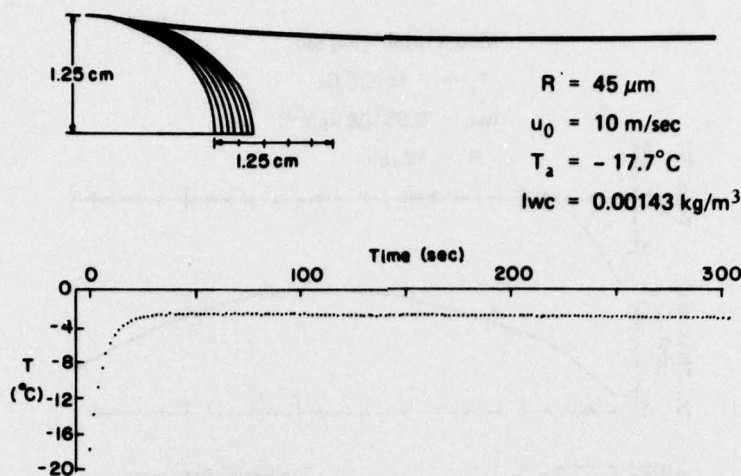
Figure 11. (Top) The change in profile dimension at 50-s intervals by ice accretion is indicated by the profile shapes on the initial half cylinder. The black line coming from the right is the tangent trajectory at the beginning (top of the black line) and end (bottom of the black line) of the icing period of 300 s. (Bottom) The surface temperature of the front half cylinder as a function of time is plotted. As seen, the surface temperature reaches an equilibrium value within about 30 s after the icing starts.

subroutine samples the distribution from the start radius r_0 to the value $r_0 + 2 \times \bar{r}$.

Histogram distribution

Dropdst also allows the specification of an arbitrary distribution such as may be obtained during experiments or in airborne sampling. In this case, the input to the subroutine is the number of droplet sizes (up to nine) and pairs of values that indicate the droplet radius and its percentage contributions to the total numbers. The subroutine then computes the volume percentages and the total number required to equal the designated lwc.

3. Option for the helicopter rotor case

As mentioned in the introduction, helicopters have serious icing problems since they normally operate at the altitudes where icing conditions most often occur. They also have the interesting characteristic that the linear velocity of the rotating blade varies over a large range (eq 1), from essentially near zero in the hub region to nearly sonic at the blade tip. Interest in how these changing conditions influence ice accretion processes was a continuing impetus in the development of this numerical scheme. Two options to include the velocity variability were therefore built into the program.

The first of these selects specific values of the velocities and computes the ice mass and profile changes

as a function of time. Figure 11 shows such a calculation for a velocity of 10 m/s and initial conditions of 0.00143 kg/m^3 lwc, droplet radius of 45 μm and $T_a = -17.7^\circ$C. Any number of these velocities can be selected, allowing details of the ice accretion process to be seen, such as the time dependence of the collection efficiency or heat transfer. The heavy black line is the tangent trajectory used to compute the collection efficiency for this case. Examples of these will be shown in the next section. The figure illustrates the three main time intervals of the calculation. The spacing of the dots in the temperature plot indicates the time period (2 s) that we take for the pseudo steady-state heat balance, i.e. in the 2-s time interval the thermodynamic properties of the system do not change. After 25 repetitions of the thermodynamic balance with successive updates of the heat flux, the profile dimensions are sufficiently changed to require the updating of the collection efficiency, ice mass and profile shape characteristics. These updates take place at 50-s intervals, as indicated by the hacks on the time scale. The changes in the profile dimensions indicated on the figure take place at these intervals. The third interval is the total time of the calculation, indicated by the full-scale length of the time scale (300 s). Programming mechanics dictate that these times be integer multiples of the smallest value. However, they can be modified within this constraint.

Model time: 300 sec

$T_a = -15.5 \; (^\circ C)$

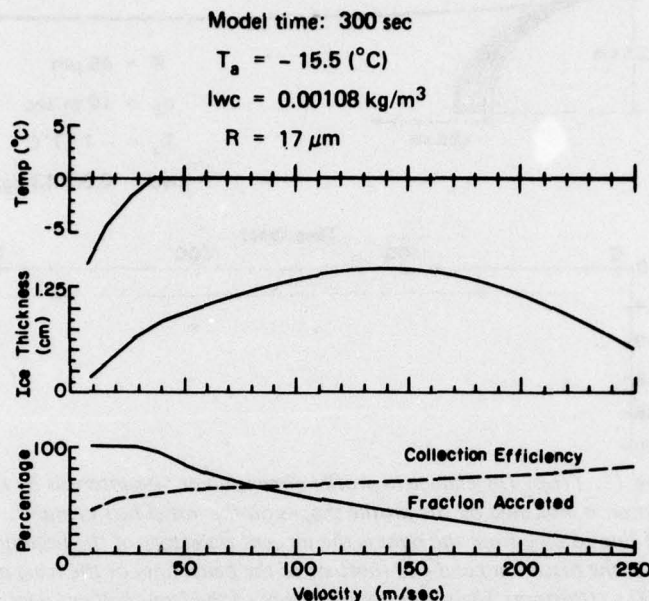$Iwc = 0.00108 \; kg/m^3$

$R = 17 \; \mu m$

Figure 12. Numerical simulation of helicopter rotor blade icing showing the surface temperature, leading edge ice thickness, collection efficiency and fraction accreted (as ice) as a function of velocity for the initial conditions as shown. The maximum ice thickness occurs when heat transfer conditions optimize with amount of accreted water (eq 6) to form the most ice.

The second display option is shown in Figure 12. Here the three quantities equilibrium surface temperature, leading edge (maximum) ice thickness, and variations of collection efficiency and fraction of collected water accreted as ice are given as a function of velocity along the blade from 0 m/s to 250 m/s. The program computes and stores the heat balance, ice thickness, and collection efficiency for each 10-m/s velocity interval for a period of 300 s and then steps to the next velocity value. When this has been accomplished for 25 velocity values (0 to 250 m/s in 10-m/s intervals), the information saved at the end of each calculational step is displayed. Straight lines are drawn between the points to complete the curves as shown. In addition, the lowest plot shows the percentage of the accreted water that is frozen into ice. As shown for this particular plot, the fraction accreted and frozen drops below 100% at the point where the surface temperature rises to 0°C as the thermodynamic equations would indicate.

As shown in Figures 11 and 12 the program also includes software for plotting data. These are given by the subroutines Plot T, Pt Blade, Plot D, Plot M and Thermopt. The function of each of these is described in the comments accompanying the program

listing in Appendix A. In addition to the plots shown in Figures 11 and 12, the program can also show a plot of the droplet size distribution if a distribution is selected. The variation of the thermodynamic quantities given in eq 7, 10, 11 and 12 as a function of segment velocity (similar to the surface temperature plot in Figure 12) can also be plotted. The quantities plotted are the convective flux, evaporative flux, kinetic energy of impact term, the latent heat requirement to freeze the accreted water, and the influx term of heat (negative) due to droplet supercooling. These are defined in eq 7, 10, 11 and 12 and discussed more fully in the following section.

RESULTS

In this section, we give some examples of the effect of including the time dependence as formulated previously. First, the effects of including collection efficiency feedback and a distribution of droplet sizes are shown in Figure 13.

Here the amount of ice accreted as time proceeds is plotted for the initial conditions of $Iwc = 0.00108$ kg/m³, initial ambient air temperature = –5.5°C, and

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Droplet radius	lwc
1	6.02 E-9
7.4	8.78 E-6
13.8	1.08 E-4
20.8	3.38 E-4
26.6	4.07 E-4
33	3.16 E-4

Gaussian Dist.

Mean 17 μ
Var 64 (8 μ S.D.)
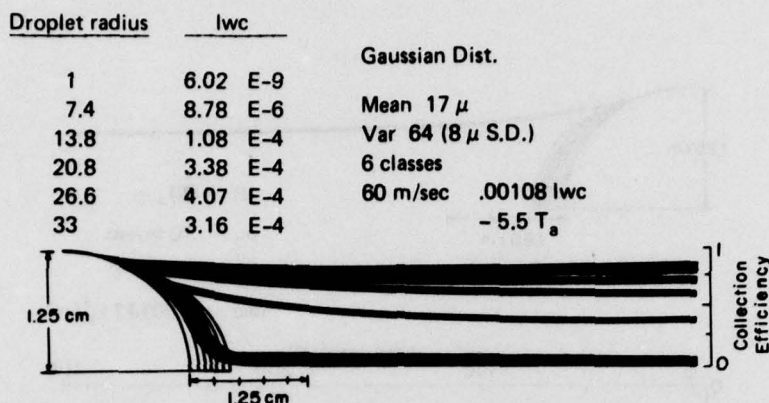6 classes
60 m/sec .00108 lwc
-5.5 T_a

Figure 13. Droplet trajectories and ice profile changes for a Gaussian droplet size distribution. Droplet sizes and respective liquid water contents are given in the inset; the collection efficiencies for the various droplet categories and their changes with profile dimension changes are given on the right hand scale. Profile ice thickness changes are shown at 50-s intervals. Thermodynamic conditions for this simulation are such that the surface temperature was at 0°C (equilibrium) and some accreted water was not frozen.

velocity = 60 m/s. The droplet distribution selected is a Gaussian with six classes. The mean radius is 17μm (17x10^{-6} m) and the droplet radius standard deviation is 8μm about the mean. As shown by the inset table listing the *lwc* amounts by droplet size, very little of the total water available is partitioned into the lower droplet sizes (for the two lowest droplet sizes the *lwc* is 2 to 3 orders of magnitude lower than for any of the four higher classes).

Referring to the profile plots, the dark lines originating at the right refer to the collection efficiency and its changes with time and droplet size as shown. The thickness of these lines results from "overprinting" on the display screen so the top of the line for any droplet class is the tangent trajectory that, when scaled to the object radius, gives the initial value of collection efficiency for that class (scale at right). The bottom of each thick line is the final collection efficiency value after 300 s of ice accretion has taken place. For the largest droplets (> 20.2μm) the collection efficiencies are generally 0.75 or above for this velocity. The collection efficiency drops markedly for sizes below 20μm and the decrease in collection efficiency with ice accretion (given by the distance between the top and bottom trajectories or thickness of each line) is also greater for the smaller droplets. The ice accretion rate, shown by the changes in the object's profile at 50-s intervals, does not change dramatically since 90% of the liquid water is concentrated in the three largest droplet classes. The collection efficiencies for these classes vary by the smallest amounts (~ 3 to 5%) at this velocity and

object radius, so the rate of ice thickness change is not substantial for the time period of this simulation. It is easily seen from this plot, however, that a lowering of the mean droplet size such that a significant shift occurs in the amount of *lwc* available in small droplets (<14μm radius) would substantially change the ice accretion rate. The collection efficiency and its change with object profile are both more markedly affected at the lower droplet sizes; for example the collection efficiency of the 1-μm category changes from 15% to 6%, a 60% change with the object profile change.

We illustrate this further in Figure 14 where two profiles are shown for the same conditions except that the droplet size is given as 30μm on the top, and half that, 15μm, on the bottom. From this figure we see that the total center line ice thickness is reduced by 40% by changing the droplet radius, even though the ambient temperature and liquid water content are the same for both cases. Similarly, the calculated surface temperature is higher for the larger droplets since more latent heat is liberated per unit time, which increases the heat flux to the object surface and increases its temperature.

A more complete treatment of the comparison beween experimental results and model simulations will be the subject of later reports. These comparisons are important because of their use in updating model results and improving on portions such as the object profile shape and heat transfer coefficient which are currently based on assumptions (e.g. the time period of steady-state thermodynamics) that may require modification. In Figure 15, we compare and discuss one such simulation with experimental data to
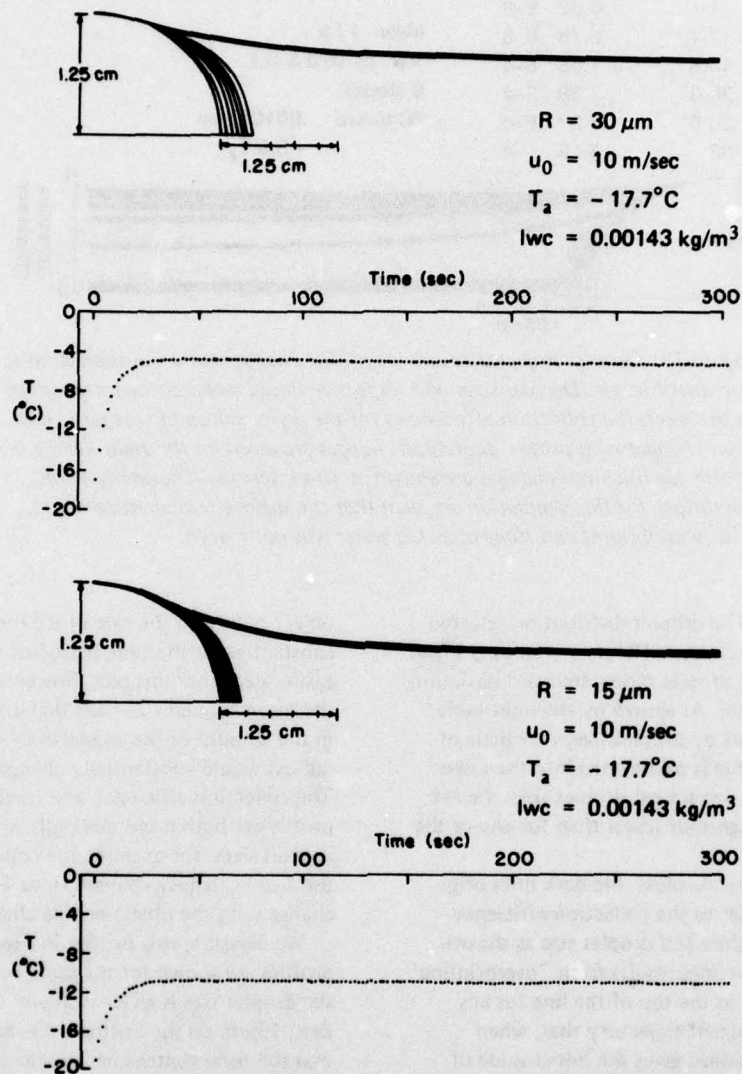
Figure 14. Accreted ice thickness and temperature for the ambient conditions indicated. The profile updates occur at 50-s intervals. The difference in accreted ice thickness for the two cases occurs because of the change in collection efficiency for the droplet sizes 30μm (top) and 15μm (bottom).

indicate both the general validity of the model and some specific differences that require more experimentation and better model parameterization. This figure, in the ice thickness distribution portion, shows the simulated and experimental results. This experimental result has been previously discussed in Ackley (1977) and is more fully described there. Briefly, the measurements were taken from a rotating cylinder

system at 3600 rpm in a coldroom at the ambient temperature shown. Water of the mean droplet size and *lwc* indicated was sprayed onto the rotor. Comparing the two ice thickness curves it is seen that the simulated ice thickness is somewhat less than the experimental accumulation but is reasonable within the constraints imposed by both the assumptions in the simulation (e.g. one droplet size at the mean

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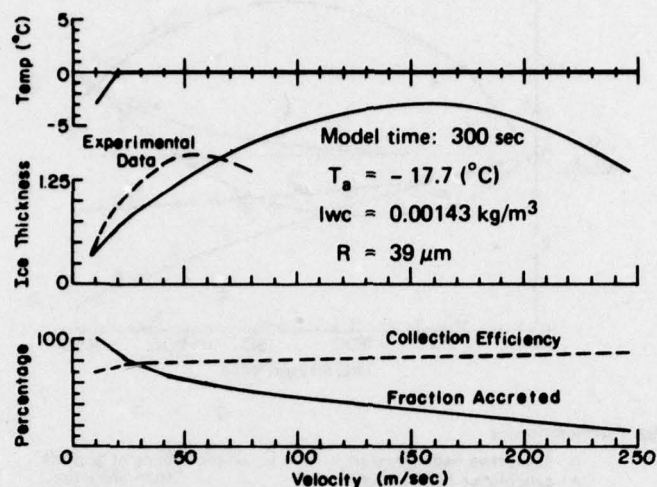
Figure 15. *Comparison between experimental data (ambient conditions inset) and model simulation of the same conditions. Agreement is reasonable considering experimental errors in lwc (~ 25%) and use of a single droplet size (mean experimental size) in the simulation compared to a distribution in the experiment.*

experimental value) and the errors in measurement in the experiment on both droplet size and *lwc* (~25%). In the experiment, analysis of the ice properties showed a grain size change, indicating a transition in surface temperature from $T < 0°C$ to $T = 0°C$ in the vicinity of the blade where the line velocity was 20 m/s. As shown by the calculated temperature distribution (top of Fig. 15) the calculated rise to 0°C takes place in about the same velocity region. This agreement would indicate the simulated heat transfer and thermodynamic relations are approximately compatible with the experimental measurements.

The location of the maximum ice thickness in the experimental data differs from that in the simulation. The latter indicates that the ice thickness should be increasing all the way to the tip of the experimental rotor ($v \sim 100$ m/s) for this set of conditions. However, a set of experiments in which the *lwc* and droplet distributions were held constant while the ambient temperature was varied showed that the location of the experimental maximum ice thickness varied only slightly with ambient temperature at roughly the same location shown in Figure 15. This continuity between the experiments indicates the maximum position may be controlled by, for example, a flow field perturbation induced by the blunt end of the cylindrical rotor. We plan further experiments at different rotation rates (i.e. changing the velocity distribution without changing the physical dimensions of the rotor) to see if and how

this effect is modified. At this time, therefore, the difference in the position of the thickness maximum between the experiments and the simulation cannot be resolved until further experimental results are available that can test the end effects of the rotor tip.

In Figure 12 we showed the accumulated ice for the conditions as indicated. Figure 16 gives the relative contributions of the thermodynamic terms for the same conditions, which we now discuss more fully. The heat balance requires that the algebraic sum of the positive and negative contributions to the heat flux equal zero as long as ice is accreting (Fig. 16). If the balance goes positive, then the excess heat is accounted for by a rise in surface temperature (eq 14). At velocities below 50 m/s, Figure 16 shows that the convective flux dominates, contributing about 50% of the necessary heat flux to freeze the accreted water, with smaller and nearly equal contributions from droplet supercooling and evaporative heat flux of approximately 25% each. At about 80 m/s, the convective flux is at its maximum negative value; it then becomes less negative with increasing velocity because of the contribution to this flux by adiabatic compressional heating of the flow ($Ru_0^2/2c_{p\,air}$ term in eq 7). Above 80 m/s, an additional positive term, the kinetic energy imparted by droplet impact, also becomes significant. The increasing positive contribution of these terms causes the amount of heat to balance
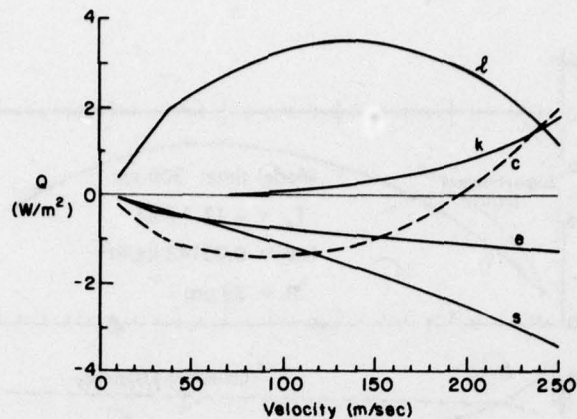
19

Figure 16. Magnitude of the individual thermodynamic terms as a
function of velocity for the simulation shown in Figure 12. The terms
are defined in Figure 9a and by eq 7, 10, 11 and 12.

the latent heat of freezing to peak and then drop off
since the other negative terms, the evaporative and
supercooling influx, decrease more slowly with
velocity than the positive terms are increasing. The
maximum available latent heat requirement corre-
sponds to the maximum ice thickness at 140 m/s
since we are in the $0°C$ surface temperature region
(Fig. 12, top). At the maximum ice thickness, the
heat necessary to freeze the accreted ice is about 45%
from droplet supercooling, 25% from evaporative flux
and 30% from convective flux. Kinetic energy of the
droplets consumes about 7% of the heat at this point,
the remainder being balanced by the latent heat of
freezing. No heat is available for freezing from con-
vective flux above 200 m/s due to the heating effects
of adiabatic compression. At about 240 m/s, the
kinetic energy of droplet impact, the now positive
convective flux, and the latent heat of freezing all
consume equal amounts of the negative flux available
from evaporative flux and droplet supercooling.

CONCLUSIONS AND FUTURE STUDIES

As pointed out earlier, one important result of the
numerical model is that a more accurate comparison
of theory with experimental results is made possible
by including time dependence in a measurable way.

These comparisons can provide more confidence in
the simulation of conditions that are not experimentally
known. For example, it is clear from Figures 13 and
14 that a change in either mean droplet size or in the
distribution such that large numbers of small droplets
compose much of the liquid water content can radically
change the amount of ice accreted. Using ground icing
sprayers with large droplet sizes for tests representing
cloud conditions may therefore be highly inaccurate.
As mass accretes the collection efficiency changes with
time and is also a function of droplet size. The numer-
ical solution graphically portrays (Fig. 13, 14) how
these conditions can affect the ice buildup for a given
set of initial conditions.

In a situation such as icing of a helicopter rotor
blade, the thermodynamics determining the ice buildup
at any particular linear velocity can show wide
variability as to which terms dominate, so a numerical
solution and graphic display can provide a convenient
method of seeing this variability (Fig. 12, 16). For
design purposes, additional terms, for example, in-
creasing the heat flux to the blade surface by electrical
heating, can be evaluated to determine whether particu-
lar icing conditions would be relieved or unaffected by
the extra heat flux. An optimization procedure could
then be used with statistics on meteorological conditions
to evaluate the likelihood of the design change signifi-
cantly increasing the chances for successful operation.

Future changes in the model will include a continuing comparison with experimental data to determine more exactly the physics controlling the icing process. Evidence exists that cross section and object shape can change considerably with changing meteorological conditions, especially at higher temperature (Dickey 1952), so a change in the way these parameters are included is probably necessary. Preliminary comparisons with experiments at temperatures below $-25°C$ (Ackley et al. 1978) indicate that the observed total ice thickness after a few minutes of accretion is less than what is predicted in the current simulation. This difference may result from a more rapid lowering of collection efficiency with ice accretion than that currently used, necessitating a reevaluation of the flow field taking into account that the object shape is irregular and not elliptical. List (1977) has also indicated that use of potential flow should be modified to include significant boundary layer effects near the object. These are not now in the model, but can be parameterized as long as experimental evidence indicates they are necessary. Finally, the most difficult parameterization involves the thermodynamics since it is the most complex and relies on the most assumptions. A major reliance is on an assumed heat transfer coefficient based on our best present knowledge. This coefficient is then used to derive the highly important surface temperature. Experiments using isotopic analysis (deuterium: hydrogen and oxygen-18:oxygen-16 ratios) on the accreted ice can perhaps be used to derive the surface temperature and independently verify our choice of heat transfer coefficient. The importance of heat flow through the back surface of the object and the variation of the heat transfer with time and conditions may then be better parameterized for model prediction use.

At present, within the data limitations that have been used for its formulation, the model offers a significant method of evaluating time dependence (or independence) of the various parameters that influence the icing process. With this information, extension to engineering design may be facilitated by easy access to a number of simulated "case histories" applying to aircraft icing or ground-based structural problems. Total icing loads can be computed by breaking the structure down geometrically, treating each shape individually, and summing up the individual loads into the total for the structure.

The beauty of the model lies in its subroutine construction, which allows updating of particular aspects (flow field, thermodynamics) as more evidence becomes available, without the need for completely altering those portions that are substantially proven.

The utility of including a more complex formulation can then be easily checked against how it changed the net result. In the present formulation, we have determined, for example, that the inclusion of a complete droplet distribution function compared to using the mean droplet size does not significantly affect the total mass accreted for most problems of engineering interest where droplet sizes exceed about 20-30μm in radius.

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```
ICE9
5   '                  *** CONTROL PROGRAM ***
10  FILE#1::'ICEFILE'
15  FILE#2::'THERMOFI'
20  FILE#3::'DROPDIST'              ' ** remember to scratch files before running **
25  DIM U(30)
30  DIM T(30)                        'velocity vector
35  DIM C(30)                        'temperature vector - used only in PT BLADE
40  DIM A(30)                        'collection efficiency vector - used only in PT BLADE
45  DIM F(30)                        'ice thickness vector - used only in PTBLADE
50  DIM X(60)                        'fraction of ice accreted vector - used only in PT BLADE
55  DIM Y(60)                        'initial x position vector - used for droplet distributions only
60  DIM E(60)                        '                           n
65  DIM W(60)                        'collection eff. vector (for individual droplets of dist.)
70  LIBRARY 'PLOTLIB***:TEK10'       'ambient lwc vector .
75  DIM M(150)
80  CALL 'BAUD':M(),1200
85  CALL 'LIMITS':M(),-1.5,-2.5,2.5
90  CALL 'PSIZE':M(),7.5,5.5
95  '
100 '                               if only the data files are to be plotted, insert here:
105 '                               ** -- LET T0= (T ambient);-- LET W= (lwc) **
110 '                               if you do not have dist. data; insert:**-- LET R=P2= (d. radius) **
                                     ** GO TO 545 **
115 LET X0= 7.5
120 LET Y0= .6
125 '       ** thermodynamic parameters appear in subprogram THERMO **
130 PRINT 'IF YOU WISH TO CHANGE ANY OF THE OBJECT PARAMETERS'
135 PRINT 'MODIFY DATA STATEMENT 300 ACCORDINGLY'
140 PRINT
145 PRINT 'DO YOU WANT TO LOOK AT A SERIES OF VELOCITY PROFILES (INPUT -1)'
150 PRINT 'OR EXAMINE THE ENTIRE BLADE DISTRIBUTION (INPUT 0)';
155 INPUT P1
160 PRINT 'DO YOU WANT TO SUPRESS ALL GRAPHICAL OUTPUT'
165 PRINT '(INPUT 1 =  SUPRESS)';
170 INPUT P4
175 IF P1>-i THEN 195
180 PRINT 'INPUT THE VELOCITY PROFILE(S) (M/S)'
185 MAT INPUT U
190 PRINT
195 PRINT 'INPUT THE LIQUID WATER CONCENTRATION (KG/M^3)'
200 PRINT 'THE AMBIENT TEMPERATURE, AND (DEG. C), AND '
205 PRINT 'THE DROPLET RADIUS (CM) (INPUT 0 IF A'
210 PRINT 'DISTRIBUTION IS DESIRED)'
215 INPUT W,T0,P2
220 LET R= P2
225 '
230 LET D= 2                        'D controls the time interval of the thermo balance - G3D must be intese
235 '
240 LET T1= 10                      'T1 sets the velocity increment size of the blade dist.
245 '
250 LET T2= 300                     'T2 sets the ** total model time **
```

```
ICE9    (continued)

255  '                                                'set up for next loop parameters
260  LET J= 250/T1
265  IF P1>-1 THEN 285
270  LET J= NUM
275  LET T1= 0
280  LET V(0)= 0
285  FOR Z= 1 TO J
290  LET U0= U(Z)+T1#Z+V(0)                           'assign U0: current velocity of balde segment.
295  READ C0,A0,B0                                     'assign : heat capacity (joules/m^des c),maj. axis (cm),minor axis (cm)
300  DATA 20,1.27,1.26                                 '   ** A0 > B0  always! otherwize elipse degenerates **
305  LET A= A0/B0                                      'dimensionless maj. axis length
310  LET B= 1                                          'dimensionless minor axis length
315  LET T= T0+ U0^2/2000                              'initialize T at recovery temperature
320  '
325  LET G3= 50                                        'G3 controls the time inteval between updations of the
330  '                                                 'coll. eff. (change 465 with G3)
335  '
340  LET M= 0                                          'reset mass accreted durring interval G3 for next profile
345  LET E= 0                                          'reset collection efficiency for next velocity profile
350  LET M0= 0                                         'reset total mass accreted
355  LET G0= 0                                         'reset model time clock
360  IF P2> 0 THEN 370                                 'are we interested in a single droplet size ?
365  CALL "DROP DST":S,X0,Y0,U0,E,E0,R,W,X(),Y(),R(),W(),E(),N   'ask what kind of distribution, and pick up first droplet size
370  CALL "PROFILE": M(),A,B,M,F,R,U0,T,P0,B0,P4,P2
375  CALL "TRAJECT": M(),X,Y,X0,Y0,U1,V1,U0,A,B,H0,R,L0,B0,F1,F4
380  CALL "COLL EFF":X,Y,X0,Y0,U1,V1,A,B,E,H0
385  IF E= 0 THEN 375                                  'have we calculated the coll eff for the drop yet ?
390  IF P2>0 THEN 410
395  CALL "DROP DST":S,X0,Y0,U0,E,E0,R,W,X(),Y(),R(),W(),E(),N   'pick up next particle size
400  IF E= 0 THEN 375                                  'if entire dist. scanned, then E = avg coll. eff., otherwize cycle back
405  LET S=1                                           'reset the droplet radius pointer so R(1) is up next
410  CALL "THERMO":E,U0,W,A,B,T0,T,F,F0,C0,M,D,Q0,A0,B0,G0,T2,G3,D
415  IF P4= 1 THEN 430                                 'plot new temperature (function of time for profile)
420  IF P1= 0 THEN 430
425  CALL "PLOT T":M(),T,G0,T2,G3,D
430  LET G0= G0+D                                      'increment model time clock D by amount of thermo balance time D
435  LET G1= G1 + M#F                                  'increment the mass accreted by the amount accreted in time D
440  IF G0<G3 THEN 410                                 'if we haven't spent enough time in the thermo balance then so back
445  LET M= G1                                         'reassign variables; let M = amount accreted in time G3,instead of D
450  LET G1= 0                                         'set total amount accreted in G3 to 0, for next cycle
455  LET M0= M0 + M                                    'M keeps track of the mass accreted over the total model time
460  IF G0= T2 THEN 485                                'check to see if we have run for the total model time yet
465  LET G3= G3+ 50                                    'calculate at what time the next profile update should occur
470  IF F= 0 THEN 410                                  'if F=0 then no ice has accreted and we needn't draw a new profile
475  LET E= 0                                          'the old E will be remembered unless we set it to zero
480  GO TO 360
485  CALL "PROFILE":M(),A,B,M,F,R,U0,T,P0,B0,P4,P2     'plot last profile of sequence
490  IF P2>0 THEN 500                                  'if a droplet dist. is not selected, then skip next line
495  PRINT #3:U0;",";E(1);",";E(2);",";E(3);",";E(4);",";E(5);",";E(6);",";E(7);",";E(8);",";E(9)
500  PRINT #1:U0;",";T;",";A;",";E;",";F              'output velocity,temp,,ice thickness,coll. eff.,and frac. accret. for pr
```

```
ICE9    (continued)
505     RESET                                               'reset data and prepare for next velocity profile
510     IF P4= 1 THEN 530
515     IF NUM= 1 THEN 555                                  'skip pause if only one profile is selected
520     CALL "MOVE":M(),-1,2.5
525     INPUT Q9                                            'this causes a **pause** between consec. profiles - input any # to cont.
530     NEXT Z
535     IF P4= 1 THEN 570
540     IF NUM= 1 THEN 555                                  'if we have data on only one profile, then no sense in plot. blade dist.
545     CALL "PT BLADE":M(),T(),C(),A(),F(),A,R,TO,W,F2,#1,T2
550     CALL "THERMOPT":M(),#2
555     IF P2> 0 THEN 565                                   'skip the plot of droplet spectrum if we have only 1 droplet size
560     CALL "PLOT D":M(),R(),E(),W(),TO,UO,W,P1,N,#3
565     CALL "FINISH":M()
570     END
575     ,
580     ,
585     ,
590     SUB "TRAJECT": M(),X,Y,XO,YO,U1,V1,UO,A,B,HO,R,LO,BO,F1,F4
595     ,  this subprogram essentially integrates newton's laws
600     ,         as applied to the droplet
605             ## UO in meters/sec   R in cm **
610     CALL "MOVE":M(),XO,YO
615     LET X=XO                                            'initialize x position of particle
620     LET Y=YO                                            ',         y
625     LET V1=U2=V2=0                                      'initialize particle velocities and accelerations
630     LET U1= -1
635     LET TO= .1                                          'TO (=t*UO/BO) is the dimmensionless time increment for the traj. calc.
640     LET K= 1340*R^2*U0*100/B0                           'calc. K - the "inirtial parameter"
645     LET LO=1                                            'stoke's law correction factor (model seems to bestcorrelate
650     ,                                   with langmuir's data when lamda = 1
655     LET RO= 12.55*100*U0*R                              'RO is the free stream particle reynolds # - not used in calc.
660     LET KO= .125 + LO*(K-.125)                          'KO is the corrected inirtial parameter (#K if LO≠1)
665             ** velocities and distances dimmensionless **
670     FOR T= 0 TO 100 STEP TO
675     CALL "VELOCITY":X,Y,U,V,A,B,W,H1,Z,F,Q,B1           'what is the free stream velocity at the droplets current position ?
680     LET R1= ((U1-U)*U0)^2+((V1-V)*U0)^2)^.5             'what is the velocity of the particle relative to the air stream ?
685     LET RO= 12.55*100*R1*R                              'what is the droplet resn. # in the moving air stream
690     IF RO> 2 THEN 705
695     LET D= 1 + 0.102*RO^0.995                           '- 695 calculates the drag coefficient based on;
700     GO TO 725                                           ', ** Beard and Prupacher's work **
705     IF RO>21 THEN 720
710     LET D= 1 + 0.115*RO^0.802
715     GO TO 725
720     LET D= 1 + .189*RO^.632
725     LET U2= -D*(U1-U)*ABS(U1-U)*TO*U0/(R1*KO)           'what is the x component of the particle acceleration
730     LET V2= -D*(V1-V)*ABS(V1-V)*TO*U0/(R1*KO)           ',        y
735     LET U1= U1+U2                                       'change in velocity due to this accelaration
740     LET V1= V1+V2
745     LET X= X+ U1*TO                                     'new position
750     LET Y= Y+ V1*TO
```

25

ICE9   (continued)

```
755     IF P4= 1 THEN 775
760     IF P1=0 THEN 775
765     IF X> 5 THEN 775
770     CALL "LINE": M(),X,Y
775     IF X>A THEN 860
780     LET Q0= B*X/(A^2*(1-(X/A)^2)^.5)        'Q0 is the profile tangent at the particles x coordinate
785     IF Q0> ABS(V1/U1) THEN 810              'compare the particle trajectory and this tangent
790     IF ABS((X/A)^2+(Y/B)^2-1)<.01 THEN 825  'check to see if particle has come close enough to call it tangent
795     ' ## trajectory missed target ##
800     LET H0= 1                               '  ## H0 takes on three values, depending on the type of traj. ##
805     GO TO 865
810     LET I= (X/A)^2 + (Y/B)^2
815     IF I>1 THEN 860
820     IF ABS(V1/U1-Q0)>.1 THEN 845
825     LET H0= 2
830     '                                       ## correct hit ##
835     '         ## trajectory within .1 arctan units of being tangent to profile ##
840     GO TO 865
845     LET H0= 3
850     '         ## particle hits target, but trajectory not tangent ##
855     GO TO 865
860     NEXT T
865     SUBEND
870     '
875     '
880     '
885     SUB "VELOCITY": X,Y,U,V,A,B,W,H1,Z,P,Q,B1    'input particle cord. (x,y), object dim. (a,b) - output free stream vel.
890     '         calculates stream velocity based on complex potential flow
895     '         theory for an elliptical cylinder
900     LET H0= (X^2+Y^2)^.5
905     LET H= ATN(Y/X)
910     LET I= ABS(H1-H)
915     IF I< 3.14/2 THEN 925
920     LET W= W+ Z*3.14159
925     LET Z= SGN(H)
930     LET H1=H
935     LET H= H + W
940     LET A0= ((X^2 -Y^2-A^2+B^2)^2+4*X^2*Y^2)^.25   '850 through 880 keeps track of atn function as it goes thru discont.
945     LET B0= (2*X*Y)/(X^2-Y^2-A^2+B^2)
950     LET B0= ATN(B0)
955     LET I= ABS(B1-B0)
960     IF I< 3.14/2 THEN 970
965     LET P= P+ Q*3.14159
970     LET Q= SGN(B0)
975     LET B1= B0
980     LET B0= .5*(B0+P)
985     LET U= (H0*B*COS(H-B0)/A0 -A)/(A-B)      '895-925 again keeps track of atn function
990     LET V= -B*H0*SIN(H-B0)/(A-B)             'output x velocity (free stream)
995     SUBEND                                    '          y
1000    '
```

26

```
1000  ICE9
1005  .  (continued)
1010  .
1015  SUB "COLL EFF"(X,Y,X0,Y0,U1,V1,A,B,E,H0    'this subprogram corrects the old particle initial position coordinates
1020  ON H0 GO TO 1025,1055,1070                   'to get a tangent trajectory based on the value of H0... or, if a
1025  .                                            tangent has been found, it assigns the coll eff a nonzero value
1030  LET S=-1
1035  LET Y1= Y0*B*((1-(X/A)^2)^.5/Y)^S
1040  IF ABS(Y0-Y1)<.001 THEN 1060                 'if the correction is less than .001, this is close enough
1045  LET Y0= Y1
1050  GO TO 1110
1055  REM THIS BRANCH FOR CORRECT HIT
1060  LET E= Y0                                    .     ** assign collection efficiency **
1065  GO TO 1110
1070  REM THIS BRANCH FOR INC. HIT
1075  LET I=V1/U1
1080  LET T= ABS(A^2*I/(B^2+(A*I)^2)^.5)
1085  LET I0= B*(1-(T/A)^2)^.5
1090  LET I= I*T+Y-I*X
1095  LET Y1= Y0*(I0/I)
1100  IF ABS(Y1-Y0)<.001 THEN 1060                 'again, if correction is less than .001, this is close enough
1105  LET Y0= Y1
1110  SUBEND
1115  .
1120  .
1125  .
1130  SUB "HTTRANS"(N,A0,B0,U0,D
1135  . THIS CORRELATION BASED ON HILPERT'S NUSSELT NUMBER
1140  . CORRELATION FOR VARIOUS FLUIDS
1145  . THIS HEAT TRANSFER RELATION
1150  . SHOULD BE ADJUSTED TO ACCOUNT
1155  . FOR THE FRONT SURFACE OF THE CYLINDER ONLY
1160  . THIS HEAT TRANSFER CORRELATION
1165  . GOOD ONLY FOR A/B < 2
1170  . AND FOR RE > 4000
1175  LET R= 12.55*U0*100*B0
1180  LET C= (.5*A0/B0+.8)/1.3                      'adjustment for elipticity
1185  LET E= .805
1190  LET K= .0239
1195  IF R> 40000 THEN 1225
1200  LET E= .618
1205  LET K= .174
1210  IF R> 4000 THEN 1225
1215  LET E= .466
1220  LET K= .615
1225  LET N= C*K*R^E                               'nusselt number based on hilpert's correlations
1230  LET N= N*.024*D*3.14159/4                    'nusselt number converted directly to α (watts/meter of blade)
1235  SUBEND
1240  .
1245  .
1250  .
```

```
1255 SUB "HTTRANS1"::N,A0,B0,T0,U0,T,D
1260   ,   THIS HEAT TRANSFER RELATION BASED ON BOSH'S CORRELATION
1265   ,   IT INCORPORATES A FILM TEMPERATURE TO DETERMINE THE
1270   ,   MASS DENSITY OF AIR, WHILE THERMAL COND.
1275   ,      DATA AND HEAT CAPACITY DATA SHOULD BE CHANGED IF
1280   ,      SIGNIFICANTLY DIFFERENT FROM VALUES AT 0 C.
1285   ,        -UNITS IN MKS-
1290 LET P= 1014                              ' SEA LEVEL PRESSURE
1295 LET F= (T0+T)/2 + 273.16
1300 LET K= .024                              'THERMAL CONDUCTIVITY OF AIR
1305   ,   heat transfer coefficient based on major axis length! watts/meter^2 deg C
1310 LET H= 31.01*(U0)^.56/(2*A0)^.44*(F*273.16/(1014*F)^^0.56
1315 LET H= 1.12BH
1320 LET NO= H*(2*A0/100)/K               'to account for assymetry of ht. transfer
1325 LET N= H*D*3.14159*(A0+B0)/(4*100)   'making an artificial transformation into the nusselt number
1330 SUBEND                               'converting directly to a (watts/meter of blade)
1335   ,  .
1340   ,  .
1345
1350 SUB "THERMO"::S,U0,W,A,B,T0,T,F,F0,CQ,M,D,Q0,A0,B0,G0,T5,#2
1355 REM TO INITIALIZE D
1360 REM THIS DOES NOT CONSIDER RADIAL FORCE EFFECTS
1365 REM NOR HEAT TRANSFER PARALLEL TO THE AXIS
1370 REM UNITS IN MKS
1375 DEF FNW(T)                               'what is the vapor pressure of water over water at T degrees
1380   LET T= T+273.16
1385   LET T1= 373.16
1390   LET I= -7.903*(T1/T -1)+5.0281*.43429*LOG(T1/T)- 1.3816*10^(-7)*(10^(11.344*( 1-T/T1))-1)
1395   LET I= I+8.1328*10^(-3)*(10^(-3.4915*(T1/T-1))-1)+.43429*LOG(1013.246)
1400   LET FNW= 10^I
1405 FNEND
1410 DEF FNI(T)                               'what is the vapor pressure of water over ice at T degrees ?
1415   LET T2= 273.16
1420   LET T= T+273.16
1425   LET I= -9.0972*(T2/T-1)-3.5665*.43429*LOG(T2/T)
1430   LET I= I+ .87679*(1-T/T2+ .43429*LOG(6.1071)
1435   LET FNI= 10^I
1440 FNEND
1445 READ R,C1,C2,P,E0,K
1450 DATA .8,1000,4180,
1455 DATA 1013,4.54,.024
1460 LET E0= FNW(T0)
1465 CALL "HTTRANS1"::N,A0,B0,T0,U0,T,D
1470 LET M= W*S*U0*(B0/100)*D               'what is the free stream vapor pressure ?
                                            'what is the heat transfer coeff. (watts/meter of blade ~ deg c ~ D sec
                                            'M is the amount of liquid water inpinging per meter of blade
1475 ON SGN(T)+2 GO TO 1635,1545,1480       'what temperature regime are we interested in ?
1480   ## this branch for T > 0 ##
1485 LET F= 0                               'fraction accreted is zero for t>o
1490 LET L0= 2500000                        'heat of vaporization of liquid water at 0 deg. c
1495 LET E= FNW(T)                          'what is the vapor pressure over the blade surface
1500 LET I0= T-T0-R*U0^2/(2*C1) +.6*L0*(E-E0)/(P*C1)
```

```
ICE9   (continued)

1505   LET I= M*C2*(T-T0-U0^2/(2*C2))        'what is the total heat transfer/meter of blade ?
1510   LET Q0= -(N*I0+I)                      'how much does this heat transfer change the surface ? temperature ?
1515   LET T1= (T*C0+Q0)/C0                   'check to see that it did not make the temp so below zero
1520   IF T1<0 THEN 1545                      (if it did, then go to t=o branch
1525   LET T=0
1530   LET T= T1
1535   GO TO 1690                             '      ** exit subprogram **
1540   ,
1545           ** this branch for T = 0 **
1550   LET T= 0
1555   LET L0= 2500000
1560   LET L= 333600                          'L is the latent heat of freezing - Joules/kg
1565   LET E= FNM(T)
1570   LET I0= -T0-R*U0^2/(2*C1)+.6*L0*(E-E0)/( C1*P)
1575   LET I= M*C2*(-T0-F*L/C2-U0^2/(2*C2))
1580   LET Q0= -(N*I0+I)
1585   LET F= F- Q0/(L*M)                     'how much more ice can be frozen (or melted) with this excess a ?
1590   LET I1= ABS(F0-F)
1595   LET F0=F.
1600   IF I1 >.001 THEN 1575
1605   IF F< 0 THEN 1480                      'if there is not enough a to freeze any ice the T goes below zero
1610   ,                                      (in this case branch to the "above zero thermodynamics")
1615   IF F>1 THEN 1635                       'if there is more than enough a to freeze the water, T drops below 0
1620   ,                                      (in this case,branch to the "below zero thermodynamics"
1625   LET C0= C0+F*M*C3                      'the accretion changes the heat capacity of the blade
1630   GO TO 1690                             '     ** exit subprogram **
1635   ,          ** this branch for T < 0 **
1640   LET F=1
1645   LET L1= 2838000                        'heat of sublimation of ice at o des. c
1650   LET L= 333600
1655   LET C3= 2356                           'heat capacity of ice
1660   LET E= FNI(T)
1665   LET I0= T-T0-R*U0^2/(2*C1) +.6*L1*(E-E0)/(P*C1)
1670   LET I= M*C2*(T*C3/C2-T0-L/C2-U0^2/(2*C2))
1675   LET Q0= -(N*I0+I)
1680   LET C0= C0+M*C3
1685   LET T= (T*C0+Q0)/C0
1690   IF (G0+2)<T5 THEN 1735                 'if this is last time increment > output thermodynamic data:
1695   LET C9= -N*(A0+B0)*3.14159/(4*100*D)*(T-T0-R*U0^2/(2*C1))  'convective heat transfer
1700   LET E9= -N*(A0+B0)*3.14159/(4*100*D)*I0 - C9              'evaporative heat transfer
1705   LET S9= -M*(A0+B0)*3.14159/(4*100*D)*C2*(T-T0)           'heating due to influx of liquid water on blade (T >= 0)
1710   IF T> 0 THEN 1720                      .  (T<0)
1715   LET S9= -M*(A0+B0)*3.14159/(4*100*D)*C2*(T*C3/C2-T0)
1720   LET K9= M*(A0+B0)*3.14159/(4*100*D)*U0^2/2               'kinetic heating of droplets on impact
1725   LET L9= -I*(A0+B0)*3.14159/(4*100*D)-S9-K9               'latent heat requirement to freeze water
1730   PRINT#2: U0;",";T;",";C9;",";E9;",";S9;",";K9;",";L9
1735   SUBEND
1740   ,
1745   ,
1750   ,
```

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(continued)

```
1735 SUB "PROFILE": M(),A,B,M,F,R,U0,T,P0,B0,P4,P2
1740 REM THIS CALCULATES  AND PLOTS THE NEW PROFILE
1745 REM  REINITIALIZE E AFTER COMPLETION
1770 CALL "ICE DEN":R,U0,T,P0,P2
1775 LET V= 1000#M/(P0#100)            'density in g/m^3
1780 LET A0= A#B0                      'volume in cm^3/cm of blade length
1785 LET A0= 4#V/(B0^2#3.14159) + A0   'old major axis length
1790 LET A= A0/B0                      'new major axis length
1795 IF P4= 1 THEN 1835
1800 LET C= A/25
1805 CALL "MOVE":M() ,0,1             'new dimensionless major axis length
1810 FOR X= 0 TO A STEP C
1815     LET Y= B#(1-(X/A)^2)^.5
1820     CALL "LINE":M(),X,Y
1825 NEXT X
1830 CALL "MOVE":M(),X0,Y0
1835 SUBEND
1840 '
1845 '
1850 '
1855 SUB "ICE DEN": R,U0,T,P0,P2
1860 REM THE DENSITY FORMULAE TAKES R IN CM
1865 IF P2= 0 THEN 1890
1870 IF T= 0 THEN 1890
1875 LET P0= .110#(10000#R#U0/ABS(T))^.76   'for droplet distributions ice density is set automatically to .917
1880 IF P0>.917 THEN 1890                    'Macklin's ice density correlation
1885 IF P0>.01 THEN 1895
1890 LET P0= .917
1895 SUBEND
1900 '
1905 '
1910 '
1915 '
1920 '
1925 '
1930 '
1935 SUB "PLOT T":M(),T,G0,T2,G3,D
1940 IF G0>0 THEN 1960
1945 CALL "MOVE":M(),0,-.5
1950 CALL "LINE":M(),0,-.5
1955 CALL "LINE":M(),4.5,-.5
1960 DEF FNY(T) = -1+(5+T)/10
1965 DEF FNX(G0)= G0#60/((12#T2)
1970 LET X= FNX(G0)
1975 LET Y= FNY(T)
1980 CALL "MOVE":M(),X,Y
1985 CALL "LINE":M(),X,Y
1990 IF G0+D<G3 THEN 2010
1995 CALL "MOVE":M(),FNX(G0),0          'put hacks on temp/time scale
2000 CALL "LINE":M(),FNX(G0),-.5
```

```
ICE9    (continued)

2005    CALL "LINE":M(),FNX(G0),-.45
2010    SUBEND
2015    .
2020    .
2025    .
2030    SUB "PT BLADE":M(),T(),C(),A(),F(),A,R,T0,W,P2,#1,T2
2035    REM THIS SUBPROGRAM PLOTS THE BLADE DISTRIBUTIONS
2040    DIM U(30)
2045    RESET #1
2050    INPUT #1:U,T,A,C,F
2055    LET K= K+1
2060    LET U(K)= U
2065    LET A(K)= A
2070    LET T(K)= T
2075    LET C(K)= C
2060    LET F(K)= F
2085    PRINT U(K),T(K),C(K),F(K)
2090    IF MORE#1 THEN 2050
2095    LET L1= 250                                 'L1 determines the graphical limit for maximum velocity
2100    CALL "CLEAR":M()
2105    CALL "CLEAR":M()
2110    CALL "LIMITS":M(),0,L1,-40,10
2115    CALL "PLIMITS":M(),1,6.5,.25,5.25
2120    FOR N= 1 TO K
2125        CALL "LINE":M(),U(N),T(N)
2130    NEXT N
2135    CALL "LIMITS":M(),0,L1,-5,5
2140    CALL "PLIMITS":M(),1,6.5,3.75,4.75
2145    CALL "AXIS":M(),0,10,0,1
2150    CALL "LIMITS":M(),0,L1,0,1
2155    CALL "PLIMITS":M(),1,6.5,.75,1.75
2160    CALL "AXIS":M(),0,10,0,.1
2165    FOR N= 1 TO K
2170        CALL "LINE":M(),U(N),C(N)
2175    NEXT N
2180    CALL "MOVE":M(),0,0
2185    FOR N= 1 TO K
2190        CALL "LINE":M(),U(N),F(N)
2195    NEXT N
2200    DEF FNA(A)= A-1
2205    CALL "PLIMITS":M(),1,6.5,2.25,3.25
2210    CALL "AXIS":M(),0,10,0,.1
2215    CALL "LIMITS":M(),0,L1,0,3
2220    CALL "PLIMITS":M(),1,6.5,2.25,5.25
2225    FOR N= 1 TO K
2230        CALL "LINE":M(),U(N),FNA(A(N))
2235    NEXT N
2240    CALL "LIMITS":M(),0,250,0,1
2245    CALL "PLIMITS":M(),0,5.5,0,5.5
2250    CALL "MOVE":M(),100,.8
```

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```
ICE9    (continued)

2255    PRINT "Temperature Distribution"
2260    CALL "MOVE":M(),130,.1
2265    PRINT "Velocity (m/sec)"
2270    CALL "MOVE":M(),250,.1
2275    PRINT .     ";L1
2280    CALL "MOVE":M(),230,.7
2285    PRINT "T = ";T0; (C)"
2290    CALL "MOVE":M(),230,.685
2295    PRINT "a"
2300    CALL "MOVE":M(),230,.655
2305    PRINT "lwc=";W;"kg/m^3"
2310    IF P2= 0 THEN 2330
2315    CALL "MOVE":M(),230,.625
2320    PRINT "R = ";R#10000; (microns)"
2325    GO TO 2355
2330    CALL "MOVE":M(),230,.625
2335    PRINT "Droplet Dist. Selected"
2340    CALL "MOVE":M(),230,.595
2345    PRINT "Avg Coll Eff Disp"
2350    CALL "MOVE":M(),100,.82
2355    CALL "MOVE":M(),0,.850
2360    PRINT .     5"
2365    PRINT
2370    PRINT "T (deg C)"
2375    PRINT
2380    PRINT .          -5"
2385    PRINT
2390    PRINT
2395    CALL "MOVE":M(),100,.56
2400    PRINT "Ice Thickness Distribution"
2405    CALL "MOVE":M(),0,.57
2410    PRINT .     1"
2415    PRINT
2420    PRINT "Thickness"
2425    PRINT .     (cm)"
2430    CALL "MOVE":M(),100,.37
2435    PRINT "Collection efficiency and Fraction"
2440    CALL "MOVE":M(),100,.34
2445    PRINT .     Accreted Distributions"
2450    CALL "MOVE":M(),0,.3
2455    PRINT .     1"
2460    CALL "MOVE":M(),230,.73
2465    PRINT "Model Time:";T2;"sec"
2470    SUBEND
2475    REM
2480    REM
2485    REM
2490    SUB "DROP DST":D,X0,Y0,U0,E,E0,R,W,X(),Y(),R(),W(),E(),N
2495    REM THIS SUB. KEEPS TRACK OF DROPLET IMPACTIONS
2500    REM SIZE, INITIAL STARTS, ECT.
```

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```
ICEP    (continued)

2305    IF D > O THEN 2785
2510    FILE#3:"DROPDIST"
2515    PRINT "INPUT 1,2,OR 3, IF A GAUSSIAN, ERLANG, OR"
2520    PRINT "SPECIFIED DISTRIBUTION IS DESIRED";
2525    INPUT A
2530    PRINT "INPUT DROPLET DATA IN MICRONS"
2535    ON A GO TO 2540,2605,2680
2540    REM THIS SAMPLES THE GAUSSIAN BETWEEN +/- 2 SIGMA OF THE MEAN
2545    REM THIS INCLUDES 95 PERCENT OF THE DISTRIBUTION
2550    PRINT "INPUT THE MEAN, VARIANCE, AND TOTAL # OF DROPLET SIZES"
2555    INPUT M,S2,N
2560    LET S= S2^.5
2565    FOR K= M-2*S TO M+2*S STEP 4*S/(N-1)
2570    IF K<O THEN 2595
2575        LET D= D+1
2580        LET P(D)= EXP(-(K-M)^2/(2*S2))/(2*3.14159*S2)^.5
2585        LET R(D)= K
2595    NEXT K
2600    GO TO 2715
2605    REM THIS BRANCH FOR ERLANG PDF
2610    REM THIS SAMPLES FROM RO TO RO + 2*MEAN
2615    PRINT "INPUT ERLANG ORDER, K*(1/L)= <MEAN>, THE DISTRIBUTION"
2620    PRINT "START RADIUS, AND THE TOTAL # OF DROPLET SIZES";
2625    INPUT K,M,RO,N
2630    LET L= K/M
2635    LET V(O)=V(1)= 1
2640    LET V(2)= 2
2645    LET V(3)= 3*V(2)
2650    FOR I= M/10 TO 2*M STEP (2*M-M/10)/(N-1)
2655        LET D= D+1
2660        LET R(D)= I+RO
2665        LET P(D)= L^K*I^(K-1)*EXP(-I*L)/V(K-1)
2670    NEXT I
2675    GO TO 2715
2680    REM THIS BRANCH FOR SPECIFIED DISTRIBUTION
2685    PRINT "INPUT # OF DROPLET SIZES IN SPECTRUM"
2690    INPUT N
2695    PRINT "INPUT DROPLET RADIUS, AND PERCENTAGE (BY NUMBERS) CONTRIBUTION"
2700    FOR K= 1 TO N
2705        INPUT R(K),P(K)
2710    NEXT K
2715    LET I= P(1)*R(1)^3+P(2)*R(2)^3+P(3)*R(3)^3+P(4)*R(4)^3+P(5)*R(5)^3
2720    LET I= I+P(6)*R(6)^3+P(7)*R(7)*R(7)^3+P(8)*R(8)^3+P(9)*R(9)^3
2725    LET H= W/(4*3.1415*I/3)
2730    PRINT "DROPLET SIZE           LWC"
2735    FOR K= 1 TO N
2740        READ X(K),Y(K)
2745        DATA 10,.7
2750        LET W(K)= 4*3.14159*R(K)^3*P(K)*H/3
2755        PRINT R(K),W(K)
```

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```
ICE9    (continued)

2760        PRINT #3:R(K);".";W(K)
2765        LET R(K)= R(K)*.0001
2770        RESET
2775    NEXT K
2780    LET D= 1
2785    LET X(D-1)= X0
2790    LET Y(D-1)= Y0
2795    LET X0= X(D)
2800    LET Y0= Y(D)
2805    LET R= R(D)
2810    LET E(D-1)= E
2815    LET E0= E0 + E*W(D-1)/W- E(D)*W(D)/W
2820    LET E= 0
2825    IF R(D)>0 THEN 2835
2830    LET E= E0
2835    LET D= D+1
2840    SUBEND
2845    REM
2850    REM
2855    REM
2860    SUB "PLOT D":M(),R(),E(),W(),T0,U0,W,P1,N,#3
2865    REM THIS SUB FOR PLOTING DROPLET DIST.
2870    RESET #3
2875    PRINT "#"
2880    INPUT Q
2885    CALL "CLEAR":M()
2890    CALL "LIMITS":M(),0,100,0,3
2895    CALL "PLIMITS":M(),2*6,.75,4.75
2900    CALL "MOVE":M(),50,2.5
2905    PRINT "Droplet Distribution"
2910    CALL "MOVE":M(),55,2.4
2915    PRINT "T = ";T0
2920    CALL "MOVE":M(),55,2.35
2925    PRINT ".a"
2930    CALL "MOVE":M(),55,2.25
2935    PRINT "U = ";U0;" m/sec"
2940    CALL "MOVE":M(),55,2.2
2945    PRINT " .o"
2950    FOR D= 1 TO N
2955        INPUT #3: R(D),W(D)
2960        CALL "LINE":M(),R(D),1000*W(D)
2965    NEXT D
2970    INPUT#3:U0,E(1),E(2),E(3),E(4),E(5),E(6),E(7),E(8),E(9)
2975    CALL "MOVE":M(),0,0
2980    FOR D= 1 TO N
2985        CALL "LINE":M(),R(D),E(D)*W(D)*1000
2990    NEXT D
2995    IF MORE#3 THEN 2970
3000    CALL "AXIS":M(),0,10,0,.25
3005    CALL "PLIMITS":M(),1.5,.25,4.25
```

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```
ICE9    (continued)

3010    CALL "MOVE":M(),0,2
3015    PRINT ' lwc'
3020    CALL "MOVE":M(),0,1.8
3025    PRINT ' (g/m^3).
3030    CALL "MOVE":M(),40,0
3033    PRINT 'Droplet Radius (microns)    100'
3040    CALL "PLIMITS":M(),1.8,5.8,.75,4.75
3045    CALL "MOVE":M(),0,2.95
3050    PRINT '3'
3055    CALL "MOVE":M(),0,1.95
3060    PRINT '2'.
3065    CALL "MOVE":M(),0,.95
3070    PRINT '1'
3075    SUBEND
3080    SUB "PLOT N":M(),I(),U()
3085    PRINT '.'.
3090    INPUT A
3095    CALL "LIMITS":M(),0,230,0,5
3100    CALL "PLIMITS":M(),1,5.5,1,5.5
3105    CALL "MOVE":M(),3,65
3110    PRINT 'Total Mass Accretion (s/cm)'
3115    CALL "AXIS":M(),0,2,0,10
3120    FOR Z= 1 TO 27
3125        IF U(Z)= 0 THEN 3140
3130        LET I(Z)= I(Z)*1000/100
3135        CALL "LINE":M(),U(Z),I(Z)
3140    NEXT Z
3145    CALL "FINISH":M()
3150    SUBEND
3155    '  .
3160    '  .
3165    '
3170    SUB "THERMOPT":M(),#2
3175    INPUT A
3180    DIM U(30),T(30),C(30),E(30),S(30),K(30),L(30)
3185    RESET #2
3190    INPUT #2:U,T,C,E,S,K,L
3195    LET N= N+1
3200    LET U(N)= U
3205    LET T(N)= T
3210    LET C(N)= C
3215    LET E(N)= E
3220    LET S(N)= S
3225    LET K(N)= K
3230    LET L(N)= L
3235    PRINT U,T,C,E,S,K,L
3240    IF MORE#2 THEN 3190
3245    CALL "CLEAR":M()
3250    CALL "CLEAR":M()
3255    LET L1= 250        'L1 sets the maximum velocity of the plot
```

ICE9   (continued)

```
3240 LET Y0= -5
3245 LET Y1= 6
3270 CALL "LIMITS":M(),0,L1,Y0,Y1
3275 CALL "PLIMITS":M(),1,5.5,.5,5
3280 CALL "AXIS":M(),0,10,0,1
3285 FOR K=1 TO N
3290    CALL "LINE":M(),U(K),C(K)
3295 NEXT K
3300 CALL "MOVE":M(),U(K),C(K)
3305 PRINT "c"
3310 CALL "MOVE":M(),0,0
3315 FOR K=1 TO N
3320    CALL "LINE":M(),U(K),E(K)
3325 NEXT K
3330 CALL "MOVE":M(),U(K),E(K)
3335 PRINT "e"
3340 CALL "MOVE":M(),0,0
3345 FOR K=1 TO N
3350    CALL "LINE":M(),U(K),S(K)
3355 NEXT K
3360 CALL "MOVE":M(),U(K),S(K)
3365 PRINT "s"
3370 CALL "MOVE":M(),0,0
3375 FOR K1=1 TO N
3380    CALL "LINE":M(),U(K1),K(K1)
3385 NEXT K1
3390 CALL "MOVE":M(),U(K1),K(K1)
3395 PRINT "k"
3400 CALL "MOVE":M(),0,0
3405 FOR K=1 TO N
3410    CALL "LINE":M(),U(K),L(K)
3415 NEXT K
3420 CALL "MOVE":M(),U(K),L(K)
3425 PRINT "l"
3430 CALL "LIMITS":M(),0,1,0,1
3435 CALL "MOVE":M(),.35,.95
3440 PRINT "Icing Thermodynamics (watts/meter^2)"
3445 CALL "MOVE":M(),7,.9
3450 PRINT "c= convective heat transfer"
3455 CALL "MOVE":M(),7,.87
3460 PRINT "e= evaporative heat transfer"
3465 CALL "MOVE":M(),7,.84
3470 PRINT "s= influx of a due to droplet."
3475 CALL "MOVE":M(),7,.81
3480 PRINT " super-cooling"
3485 CALL "MOVE":M(),7,.78
3490 PRINT "k= kinetic heating of droplets"
3495 CALL "MOVE":M(),7,.75
3500 PRINT " from impaction"
3505 CALL "MOVE":M(),7,.72
```

'Y0 sets minimum on heat transfer scale
'Y1 sets maximum on heat transfer scale

ICE9   (continued)

```
3510 PRINT "l= latent heat requirement to"
3515 CALL "MOVE":M(),7,.69
3520 PRINT " freeze accreted water."
3525 CALL "MOVE":M(),.4,-Y0/(Y1-Y0)-.04  ;L1
3530 PRINT "velocity (m/sec)"
3535 CALL "MOVE":M(),0,1
3540 PRINT
3545 PRINT Y1
3550 CALL "MOVE":M(),0,.1
3555 PRINT
3560 PRINT Y0
3565 subend
```

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